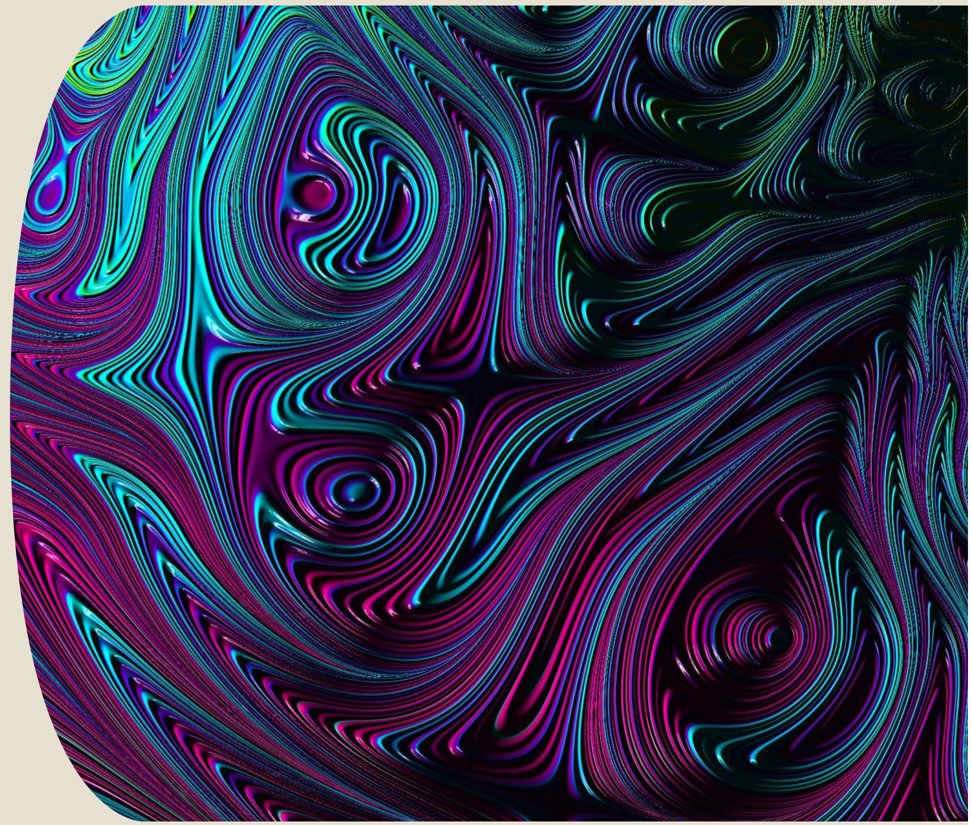
TEXAS A&M UNIVERSITY

# Advanced Numerical Methods in the Mathematical Sciences



**May 4–7, 2015**

**Texas A&M University**

**College Station, Texas**

**Rudder Tower, Room 501**

## Acknowledgements

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College of Science at Texas A&M

Institute for Applied Mathematics and Computational Science at Texas A&M

Institute for Scientific Computation at Texas A&M


## Day 1: Monday, May 4, 2015

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|---|---|
| 8–8:45 a.m. | Registration |
| 8:45–9 a.m. | Welcoming Remarks |
| 9–9:45 a.m. | Mary Wheeler, University of Texas at Austin<br>*Modeling Coupled Flow, Chemistry, and Mechanics in Porous Media* |
| 9:45–10:30 a.m. | Konstantin Lipnikov, Los Alamos National Laboratory<br>*The Mimetic Finite Difference and Virtual Element Methods* |
| 10:30–11 a.m. | Break |
| 11–11:45 a.m. | Johnny Guzmán, Brown University<br>*Higher-Order Finite Element Methods for Elliptic Problems with Interface* |
| 11:45 a.m.–1:30 p.m. | Lunch Break |
| 1:30–2:15 p.m. | Junping Wang, National Science Foundation<br>*Weak Galerkin Finite Element Methods for* **div-curl** *Systems* |
| 2:15–3 p.m. | Lin Mu, Michigan State University<br>*Weak Galerkin Finite Element Methods and Numerical Applications* |
| 3–3:30 p.m. | Break |
| 3:30–6 p.m. | Poster Session<br>Memorial Student Center, Room 2404 |

**Mary Wheeler, University of Texas at Austin**
*Modeling Coupled Flow, Chemistry, and Mechanics in Porous Media*

We describe realistic physical models for modeling coupled flow, mechanics, and geochemistry in porous media. Robust algorithms for treating relevant applications are discussed and computational results presented.

**Konstantin Lipnikov, Los Alamos National Laboratory**
*The Mimetic Finite Difference and Virtual Element Methods*

The mimetic finite difference (MFD) method mimics fundamental properties of mathematical and physical systems such as conservation laws, duality and self-adjointness of differential operators, and exact mathematical identities of the vector and tensor calculus. The MFD method works on general polygonal and polyhedral meshes. The virtual element method (VEM) was introduced recently as an evolution of the nodal MFD method.

In the first part of this talk, I'll review common underlying concepts of both methods, with the focus on consistency and stability conditions and efficient computer implementation of the methods. In the second part of the talk, I'll show how flexibility of the MFD framework can be used to design new schemes with special properties for nonlinear evolution problems.

More precisely, I'll explain how upwinding of problem coefficients can be included in mimetic schemes.

**Johnny Guzmán, Brown University**
*Higher-Order Finite Element Methods for Elliptic Problems with Interface*

Co-Authors: Manuel Sanchez-Uribe and Marcus Sarkis

We present higher-order piecewise continuous finite element methods for solving a class of interface problems in two dimensions. The method is based on correction terms added to the right-hand side in the standard variational formulation of the problem. We prove optimal error estimates of the methods on general quasi-uniform and shape regular meshes in maximum norms. In addition, we apply the method to a Stokes interface problem, adding correction terms for the velocity and the pressure, obtaining optimal convergence results.

**Eric Chung, Chinese University of Hong Kong**
*Adaptive Multiscale Discontinuous Galerkin Methods*

In this talk, we present an adaptive Generalized Multiscale Discontinuous Galerkin Method (GMsDGM) for a class of high-contrast flow problems, and derive a-priori and a-posteriori error estimates for the method. Based on the a-posteriori error estimator, we develop an adaptive enrichment algorithm for our GMsDGM and prove its convergence. The adaptive enrichment algorithm gives an automatic way to enrich the approximation space in regions where the solution requires more basis functions, which are shown to perform well compared with a uniform enrichment. The proposed error indicators are $L_2$-based and can be inexpensively computed which makes our approach efficient. Numerical results are presented that demonstrate the robustness of the proposed error indicators.

**Alan Demlow, Texas A&M University**
*A Posteriori Error Estimation and Adaptivity in the Maximum Norm*

Co-Author: Natalia Kopteva

In this talk, I will discuss recent progress in constructing adaptive algorithms for controlling errors in the maximum norm. The main focus will be the construction of robust a posteriori error estimators for singularly perturbed elliptic reaction-diffusion problems. I will also discuss the sharpness of logarithmic factors that typically arise in maximum norm a posteriori estimates. In particular, we show that standard estimators for controlling maximum errors efficiently and reliably control the error in a modified bounded mean oscillation (BMO) norm with no logarithmic factors present.

**Ivan Yotov, University of Pittsburgh**

*Domain Decomposition and Time-Partitioned Methods for Flow in Fractured Poroelastic Media*

We discuss a computational framework for modeling multiphysics systems of coupled flow and mechanics problems. The simulation domain is decomposed into a union of subdomains, each one associated with a physical, mathematical, and numerical model. Physically meaningful interface conditions are imposed on the discrete level via mortar finite elements or Nitsche's coupling. We present applications of the framework to modeling flow in fractured poroelastic media and arterial flows based on Navier Stokes/Stokes/Brinkman flows coupled with the Biot system of poroelasticity. We discuss stability and accuracy of the spatial discretizations and loosely coupled non-iterative time-split formulations. We further study the use of the loosely coupled scheme as a preconditioner for the monolithic scheme and establish a spectral equivalence of the two formulations. A reduced-dimension fracture model will also be discussed.

**Ludmil Zikatanov, Pennsylvania State University and Bulgarian Academy of Sciences**

*Stability and Monotonicity in the Low Order Discretizations of the Biot's Model*

Co-Authors: C. Rodrigo, F.J. Gaspar, and X. Hu

We consider finite element discretizations of the Biot's model in poroelasticity with lowest order (MINI and stabilized P1-P1) elements. We show convergence of discrete schemes which are implicit in time and use these types of elements in space. We deal with 1, 2, and 3 spatial dimensions in a unified fashion. We also address the issue related to the presence of non-physical oscillations in the pressure approximations for low permeabilities and/or small time steps. We show that even in 1D a Stokes-stable finite element pair does not provide a monotone discretization for low permeabilities. We then introduce a stabilization term which removes the oscillations. We present numerical results confirming the monotone behavior of the stabilized schemes.

**Junping Wang, National Science Foundation**

*Weak Galerkin Finite Element Methods for* **div-curl** *Systems*

Co-Author: Chunmei Wang

This talk shall introduce a new numerical technique, called the weak Galerkin finite element method (WG), for partial differential equations. The presentation will start with the second-order elliptic equation, for which WG shall be applied and explained in detail. The concept of weak gradient will be introduced and discussed for its role in the design of weak Galerkin finite element schemes. The speaker will then introduce a general notion of weak differential operators, such as weak Hessian, weak divergence, and weak **curl**, etc. These weak differential operators shall serve as building blocks for WG finite element methods for other classes of partial differential equations, such as the Stokes equation, the biharmonic equation for thin plate bending, the Maxwell equations in electron magnetics theory, and the **div-curl** systems. In particular, the speaker will demonstrate how WG can be applied to the **div-curl** systems. A mathematical convergence theory shall be briefly discussed for this application. The talk should be accessible to graduate students with adequate training in computational methods.

**Lin Mu, Michigan State University**

*Weak Galerkin Finite Element Methods and Numerical Applications*

Weak Galerkin finite element methods are new numerical methods for solving partial differential equations (PDEs) that were first introduced by Wang and Ye for solving general second-order elliptic PDEs. The differential operators in PDEs are replaced by their weak forms through integration by parts, which endows high flexibility for handling complex geometries, interface discontinuities, and solution singularities. This new method is a discontinuous finite element algorithm, which is parameter free, symmetric, and absolutely stable. Furthermore, through the Schur-complement technique, an effective implementation of the weak Galerkin is developed as a linear system involving unknowns only associated with element boundaries. In this talk, several numerical applications of weak Galerkin methods will be discussed.

## Day 2: Tuesday, May 5, 2015

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|---|---|
| 8:30–9:15 a.m. | Bernardo Cockburn, University of Minnesota<br>*The Evolution of the HDG Methods: An Overview* |
| 9:15–10 a.m. | Weifeng Qiu, City University of Hong Kong<br>*Robust A Posteriori Error Estimates for the HDG Method for Convection-Diffusion Equations* |
| 10–10:30 a.m. | Break |
| 10:30–11:15 a.m. | Melvin Leok, University of California at San Diego<br>*Space-Time Finite-Element Exterior Calculus and Variational Discretizations of Gauge Field Theories* |
| 11:15 a.m.–noon | Yanqiu Wang, Oklahoma State University<br>*In Search of Conforming* **$H$(curl)** *and* **$H$(div)** *Elements on Polytopes* |
| noon–1:30 p.m. | Lunch Break |
| 1:30–2:15 p.m. | Tan Bui-Thanh, University of Texas at Austin<br>*Some Recent Advances in Hybridized Discontinuous Galerkin Methods* |
| 2:15–3 p.m. | Guo-Wei Wei, Michigan State University<br>*Objective-Oriented Multidimensional Persistence in Biomolecular Data* |
| 3–3:30 p.m. | Break |
| 3:30–5:30 p.m. | Leszek Demkowicz, University of Texas at Austin<br>*Discontinuous Petrov-Galerkin (DPG) Method with Optimal Test Functions: Tutorial and Perspectives* |

### Jay Gopalakrishnan, Portland State University

*Traces of a Friedrichs Space for the Wave Equation and Tent Pitching*

The time-dependent wave equation can be regarded as a Friedrichs system. The wave operator then becomes a continuous operator on a Hilbert space normed with a graph norm. In this talk, focusing on the case of one space dimension, we discuss a trace theory for this space. In order to set up a weak formulation using the modern theory of Friedrichs systems, one must incorporate the boundary conditions into an intrinsic double cone within this space. This is done using the trace theory. Of particular interest are domains whose inflow and outflow boundaries intersect. Such domains arise in explicit finite element schemes of the tent pitching type. We find that the traces have a weak continuity property at the meeting points of the inflow and outflow boundaries. Motivated by this continuity property, we use discrete spaces that conform to the continuity property, thus resulting in a new explicit scheme with locally variable time and space mesh sizes.

### Joachim Schöberl, Vienna Institute of Technology

*DG and HDG Tent Pitching Methods for Conservation Laws*

Co-Authors: Jay Gopalakrishnan and Christoph Wintersteiger

We present a new approach to discretize hyperbolic conservation laws in space-time by local solvers on patches. In contrast to earlier tent-pitching methods which are based on full 4D discretizations, our method recycles usual discontinuous or hybrid discontinuous Galerkin methods in space combined with traditional time-stepping methods. Numerical results for the linear wave equation and non-linear equations are presented.

## Day 4: Thursday, May 7, 2015

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|---|---|
| 8:30–9:15 a.m. | Jay Gopalakrishnan, Portland State University<br>*Traces of a Friedrichs Space for the Wave Equation and Tent Pitching* |
| 9:15–10 a.m. | Joachim Schöberl, Vienna Institute of Technology |
| 10–10:30 a.m. | Break |
| 10:30–11:15 a.m. | Ivan Yotov, University of Pittsburgh<br>*Domain Decomposition and Time-Partitioned Methods for Flow in Fractured Poroelastic Media* |
| 11:15 a.m.–noon | Ludmil Zikatanov, Pennsylvania State University and Bulgarian Academy of Sciences<br>*Stability and Monotonicity in the Low Order Discretizations of the Biot's Model* |
| noon–2 p.m. | Lunch Break |
| 2–2:45 p.m. | Eric Chung, Chinese University of Hong Kong<br>*Adaptive Multiscale Discontinuous Galerkin Methods* |
| 2:45–3:30 p.m. | Alan Demlow, Texas A&M University<br>*A Posteriori Error Estimation and Adaptivity in the Maximum Norm* |
| 3:30–3:45 p.m. | Closing Remarks |

### Bernardo Cockburn, University of Minnesota

*The Evolution of the HDG Methods: An Overview*

We describe the evolution of the so-called hybridizable discontinuous Galerkin (HDG) since their inception (back in 2005) until the present day. After defining them, within the setting of steady-state diffusion, we present several ways of rewriting the methods to be able to establish links with many other numerical methods, especially with the discontinuous Galerkin methods. We then sketch the main developments of the HDG methods. We end by briefly describing the main developments of their applications to a wide variety of partial differential equations of practical interest.

### Weifeng Qiu, City University of Hong Kong

*Robust A Posteriori Error Estimates for the HDG Method for Convection-Diffusion Equations*

We propose a robust a posteriori error estimator for the hybridizable discontinuous Galerkin (HDG) method for convection-diffusion equations with dominant convection. The reliability and efficiency of the estimator are established for the error measured in an energy norm. The energy norm is uniformly bounded even when the diffusion coefficient tends to zero. The estimators are robust in the sense that the upper and lower bounds of error are uniformly bounded with respect to the diffusion coefficient. A weighted test function technique and the Oswald interpolation are key ingredients in the analysis. Numerical results verify the robustness of the proposed a posteriori error estimator. In numerical experiments, optimal convergence is observed.

**Melvin Leok, University of California at San Diego**

*Space-Time Finite-Element Exterior Calculus and Variational Discretizations of Gauge Field Theories*

Many gauge field theories can be described using a multisymplectic Lagrangian formulation, where the Lagrangian density involves space-time differential forms. While there has been prior work on finite-element exterior calculus for spatial and tensor product space-time domains, less has been done from the perspective of space-time simplicial complexes. One critical aspect is that the Hodge star is now taken with respect to a pseudo-Riemannian metric, and this is most naturally expressed in space-time adapted coordinates, as opposed to the barycentric coordinates that Whitney forms are typically expressed in terms of.

We introduce a novel characterization of Whitney forms and their Hodge dual with respect to a pseudo-Riemannian metric that is independent of the choice of coordinates, and then apply it to a variational discretization of the covariant formulation of Maxwell's equations. Since the Lagrangian density for this is expressed in terms of the exterior derivative of the four-potential, the use of finite-dimensional function spaces that respects the de Rham cohomology results in a discretization that inherits the gauge symmetries of the continuous problem. This yields a variational discretization that exhibits a discrete Noether's theorem.

**Yanqiu Wang, Oklahoma State University**

*In Search of Conforming* $\boldsymbol{H}(\mathbf{curl})$ *and* $\boldsymbol{H}(\mathbf{div})$ *Elements on Polytopes*

We construct $\boldsymbol{H}(\mathbf{curl})$ and $\boldsymbol{H}(\mathbf{div})$ finite elements on convex polygons and polyhedra. These elements can be viewed as extensions of the lowest order Nedelec-Raviart-Thomas elements, from simplices to general convex polytopes. The construction is based on generalized barycentric coordinates and the Whitney forms. In 3D, it currently requires the faces of the polyhedron be either triangles or parallelograms. Unified formula for computing basis functions are given. The finite elements satisfy discrete de Rham sequences in analogy to the well-known ones on simplices. Moreover, they reproduce existing $\boldsymbol{H}(\mathbf{curl})$ and $\boldsymbol{H}(\mathbf{div})$ elements on simplices, parallelograms, parallelepipeds, pyramids, and triangular prisms. Approximation property of the constructed elements is obtained on arbitrary convex polygons and certain polyhedra.

**Seong Lee, Chevron Energy Technology Company**

*Recent Advances in Numerical Methods Applied to Non-Linear Transport Equations in Reservoir Simulation*

This paper discusses modeling and mathematical challenges in developing efficient, accurate numerical methods to solve multi-phase flow in heterogeneous porous media. The major mathematical difficulties originate from the uncertainties in governing equations and boundary geometries and scale-dependent complexity in physical parameters and measurements. For instance, the permeability of natural formations displays high variability levels and complex structures of spatial heterogeneity which spans a wide range of length scales. Relative permeabilities for multi-phase flow are measured in a laboratory by a core sample of 1-2 inches, and they are directly applied in a reservoir simulation grid of 10-100 feet that may include large and different heterogeneity. This paper reviews recent advances in numerical methods for non-linear transport equations in reservoir simulation:

- Multi-point flux approximation.
- Hierarchical approach to naturally fractured reservoir with multiple length scales.
- Multi-scale finite volume method.
- Dynamic upscaling/downscaling.
- Sequential fully implicit method.

The paper also reviews issues and challenges in practical simulation of field scale models.

**Randolph Bank, University of California at San Diego**

*Some Recent Results for Adaptive Finite Elements*

We will discuss our ongoing investigation of adaptive strategies for finite element equations. We show first that under modest assumptions, any robust and efficient a posteriori error indicator must behave as a simple interpolation error for the exact finite element solution. We then use the interpolation error to study several popular $h$ and $hp$ adaptive algorithms.

**Todd Arbogast, University of Texas at Austin**

*Approximation of a Degenerate Elliptic Equation Arising from a Two-Phase Mixture Modeling the Motion of the Earth's Mantle*

Co-Authors: Marc A. Hesse and Abraham L. Taicher

We consider the linear but degenerate elliptic system of two first-order equations $\mathbf{u} = -\phi \nabla p$ and $\nabla \cdot (\phi \mathbf{u}) + \phi p = \phi f$, where the porosity $\phi \geq 0$ may be zero on a set of positive measure. The model equation we consider has a similar degeneracy as that arising in the equations describing the mechanical system modeling the dynamics of partially melted materials, e.g., in the Earth's mantle, and the flow of ice sheets, e.g., in the polar ice caps and glaciers. In the context of mixture theory, $\phi$ represents the phase variable separating the solid one-phase $(\phi = 0)$ and fluid-solid two-phase $(\phi > 0)$ regions. Two main problems arise. First, as $\phi$ vanishes, one equation is lost. Second, after we extract stability or energy bounds for the solution, we see that the pressure $p$ is not controlled outside the support of $\phi$. After an appropriate scaling of the pressure, we can show existence and uniqueness of a solution over the entire domain. We then develop a stable mixed finite element method for the problem, and show some numerical results.

**Tan Bui-Thanh, University of Texas at Austin**

*Some Recent Advances in Hybridized Discontinuous Galerkin Methods*

We will present several new developments on the emerging Hybridized Discontinuous Galerkin (HDG) method. First, starting either from the Godunov upwind idea or from the Rankine-Hugoniot condition we derive a unified HDG framework for linear PDEs that allows one to uncover new HDG methods and recover most of the existing ones for a large class of PDE including the Friedrichs' systems. Analysis and numerical results for the unified framework will be presented. Second, we will present an IMEX scheme for the HDG method with application to atmospheric sciences. Third, we will present a multilevel HDG solver that is promising to be one of the fast and parallel solvers for large-scale problem. Fourth, we will present our work on parallel $hp$ method for HDG methods. Finally, a non-conforming HDG will be introduced and analyzed.

**Guo-Wei Wei, Michigan State University**

*Objective-Oriented Multidimensional Persistence in Biomolecular Data*

Geometric apparatuses are frequently inundated with too much structural detail to be computationally tractable, while traditional topological tools often incur too munch simplification of the original data to be practically useful. Persistent homology, a new branch of algebraic topology, is able to bridge the gap between geometry and topology. In this talk, I will discuss a few new developments in persistent homology. First, we introduce multiscale persistent homology to describe the topological fingerprints and topological transitions of macromolecules. Additionally, multidimensional persistence is developed for topological denoising and revealing the topology-function relationship in biomolecular data. Moreover, molecular topological fingerprints are utilized to resolve ill-posed inverse problems in cryo-EM structure determination. Finally, objective-oriented persistent homology is constructed via the variational principle and differential geometry for proactive feature extracting from big data sets, which leads to topological partial differential equations (TPDEs).

**Leszek Demkowicz, University of Texas at Austin**

*Discontinuous Petrov-Galerkin (DPG) Method with Optimal Test Functions: Tutorial and Perspectives*

Co-Author: Jay Gopalakrishnan, Portland State University

The DPG method wears three hats [2]. It is a Petrov-Galerkin method with optimal test functions computed on the fly to reproduce stability properties of the continuous variational formulation [1]. It is also a minimum-residual method with the residual measured in a dual norm corresponding to a user-defined test norm [1,2]. This gives (an indirect) possibility of controlling the norm of convergence, a feature especially attractive in context of constructing robust discretizations for singular perturbation problems [4,5,9]. Finally, it is also a mixed method in which one simultaneously solves for the residual [3,2], which provides an excellent a-posteriori error estimator and enables automatic adaptivity.

In practice, the optimal test functions and residual are approximated within a finite-dimensional enriched test space. The mixed method framework provides a natural starting point for analyzing effects of such an approximation through the construction of appropriate Fortin operators [7,4].

What makes the whole story possible is the use of discontinuous test functions (broken test spaces). The trial functions may, but need not be, discontinuous. The paradigm of "breaking" test functions can be applied to any well-posed variational formulation [8,4] including well known classical and mixed formulations and the less known ultraweak formulation. When applied to trivial (strong) variational formulation, DPG reduces to the well known least-squares method. DPG results in a hybridization of the original formulation where one solves additionally for fluxes (and traces in the ultraweak case) on the mesh skeleton. The hybridization approximately doubles the number of interface unknowns when compared with classical conforming elements or HDG methods but it is comparable with a number of unknowns for other DG formulations. Computation of optimal test functions and residual is done locally, at the element level. Thus, it does not contribute to the cost of the global solve (but it is significant for systems of 3D equations).

The DPG methodology guarantees a stable discretization for any well-posed boundary- or initial boundary-value problem (space-time formulations). In particular, it can be applied to all problems where the standard Galerkin fails

**Victor Calo, King Abdullah University of Science and Technology**

*PetIGA: High-Performance Isogeometric Analysis*

Co-Authors: N.O. Collier, A.M.A. Cortes, L.A. Dalcin, A. Sarmiento, and P. Vignal

We have developed fast implementations of B-spline/NURBS based finite element analysis, written using PETSc. PETSc is frequently used in software packages to leverage its optimized and parallel implementation of solvers; however, we also are using PETSc data structures to assemble the linear systems. These structures were originally intended for the parallel assembly of linear systems resulting from finite differences. We have reworked this structure for linear systems resulting from isogeometric analysis based on tensor product spline spaces. The result of which is the PetIGA framework for solving problems using isogeometric analysis which is scalable and greatly simplified over previous solvers.

Our infrastructure has also allowed us to develop scalable solvers for a variety of problems. We have chosen to pursue nonlinear time dependent problems, such as:

- Cahn-Hilliard
- Navier-Stokes-Korteweg
- Variational multiscale for Navier-Stokes
- Diffusive wave approximation to shallow water equations
- Phase-field crystal equation and its time integration
- Divergence-conforming B-spline model for nanoparticle suspensions

We also have solvers for an assortment of linear problems: Poisson, elasticity, Helmholtz, thin shells, advection-diffusion, and diffusion-reaction. All solvers are written to be inherently parallel and run on anything from a laptop to a supercomputer such as Shaheen, KAUST IBM-BlueGeneP supercomputer. In this presentation, we will focus on new time integration techniques for phase-field modeling which are energy stable and allow for stable linearizations of the underlying non-linear model as well as on divergence conforming discretizations for nanoparticle suspensions.

**Pavel Bochev, Sandia National Laboratories**

*Optimization-Based Coupling of Local and Non-Local Continuum Models: A Divide and Conquer Approach for Stable and Physically Consistent Heterogeneous Numerical Models*

We formulate and analyze an optimization-based method for coupling of local continuum models such as Partial Differential Equation (PDE), with nonlocal continuum descriptions in which interactions can occur at distance, without contact. Examples of the latter include nonlocal continuum mechanics theories such as peridynamics or physics-based nonlocal elasticity, which can model pervasive material failure and fracture and can also result from homogenization of nonlinear damage models.

The purpose of such Local-to-Nonlocal (LtN) couplings is to combine the computational efficiency of PDEs with the accuracy of nonlocal models, which can incorporate strong nonlocal effects due to long-range forces at the mesoscale or microscale. The need for local-nonlocal couplings is especially acute when the size of the computational domain is such that the nonlocal solution becomes prohibitively expensive to compute, yet the nonlocal model is required to accurately resolve small scale features such as crack tips or dislocations that can affect the global material behavior.

The main idea of the presented approach is to couch the coupling of the two models into an optimal control problem where the states are the solutions of the nonlocal and local equations, the objective is to minimize their mismatch on the overlap of the local and nonlocal problem domains and the controls are the nonlocal volume constraint and the local boundary condition. We present the method in the context of local-to-nonlocal diffusion coupling. Numerical examples in one-dimension illustrate the theoretical properties of the approach.

or is stable only in the asymptotic regime. Being a Ritz method, DPG enables adaptive computations starting with coarse meshes. For instance, for wave propagation problems, the initial mesh need not even satisfy the Nyquist criterion.

The presentation will consist of two parts. First, the tutorial will provide an overview of the main points made above, illustrated with 1D and 2D numerical examples. The second part will outline some open problems and some of our current work on the subject.

**References:**

1. L. Demkowicz and J. Gopalakrishnan, *A Class of Discontinuous Petrov-Galerkin Methods. Part II: Optimal Test Functions*, Num. Meth. Part. D.E. (2011) 27: 70-105.
2. L. Demkowicz and J. Gopalakrishnan, *An Overview of the DPG Method, in Recent Developments in Discontinuous Galerkin Finite Element Methods for Partial Different Equations*, eds: X. Feng, O. Karakashian, Y. Xing, IMA Publications, Springer-Verlag (2013).
3. A. Cohen, W. Dahmen, and G. Welper, *Adaptivity and Variational Stabilization for Convection-Diffusion Equations*, ESAIM Math. Model. Numer. Anal. (2012), 46(5): 1247-1273.
4. L. Demkowicz, C. Carstensen, and J. Gopalakrishnan, *The Paradigm of Broken Test Functions in DPG Discretizations*, in preparation.
5. L. Demkowicz and H. Heuer, *Robust DPG Method for Convection-Dominated Diffusion Problems*, SIAM J. Num. Anal. (2013), 51: 2514-2537.
6. J. Chan, N. Heuer, T. Bui-Thanh, and L. Demkowicz, *Robust DPG Method for Convection-Dominated Diffusion Problems II: Natural Inflow Condition*, Comput. Math. Appl. (2014), 67(4): 771-795.
7. J. Gopalakrishnan and W. Qiu, *An Analysis of the Practical DPG Method*, Math. Comp. (2014) 83(286): 537-552.
8. L. Demkowicz, *Various Variational Formulations and Closed Range Theorem*, ICES Report 15-03.
9. L. Demkowicz and I. Harari, *Robust Discontinuous Petrov Galerkin (DPG) Methods for Reaction-Dominated Diffusion*, ICES Report 14-36.

## Day 3: Wednesday, May 6, 2015

| | |
|---|---|
| 8:30–9:15 a.m. | Chi-Wang Shu, Brown University<br>*IMEX Time Marching for Discontinuous Galerkin Methods* |
| 9:15–10 a.m. | Beatrice Riviere, Rice University<br>*High Order Methods for Flows in Heterogeneous Porous Media* |
| 10–10:30 a.m. | Break |
| 10:30–11:15 a.m. | Pavel Bochev, Sandia National Laboratories<br>*Optimization-Based Coupling of Local and Non-Local Continuum Models: A Divide and Conquer Approach for Stable and Physically Consistent Heterogeneous Numerical Models* |
| 11:15 a.m.–noon | Victor Calo, King Abdullah University of Science and Technology<br>*PetIGA: High-Performance Isogeometric Analysis* |
| noon–1:30 p.m. | Lunch Break |
| 1:30–2:15 p.m. | Randolph Bank, University of California at San Diego<br>*Some Recent Results for Adaptive Finite Elements* |
| 2:15–3 p.m. | Todd Arbogast, University of Texas at Austin<br>*Approximation of a Degenerate Elliptic Equation Arising from a Two-Phase Mixture Modeling the Motion of the Earth's Mantle* |
| 3–3:30 p.m. | Break |
| 3:30–5:30 p.m. | Seong Lee, Chevron Energy Technology Company<br>*Recent Advances in Numerical Methods Applied to Non-Linear Transport Equations in Reservoir Simulation* |
| 6:15–9:30 p.m. | Dinner<br>Café Eccell<br>4401 S. Texas Avenue, Bryan, TX 77802 - 979.599.7929 |

### Chi-Wang Shu, Brown University

*IMEX Time Marching for Discontinuous Galerkin Methods*

Co-Authors: Haijin Wang, Qiang Zhang, and Yunxian Liu

For discontinuous Galerkin methods approximating convection diffusion equations, explicit time marching is expensive since the time step is restricted by the square of the spatial mesh size. Implicit methods, however, would require the solution of non-symmetric, non-positive definite and nonlinear systems, which could be difficult. The high order accurate implicit-explicit (IMEX) Runge-Kutta or multi-step time marching, which treats the diffusion term implicitly (often linear, resulting in a linear positive-definite solver) and the convection term (often nonlinear) explicitly, can greatly improve computational efficiency. We prove that certain IMEX time discretizations, up to third-order accuracy, coupled with the local discontinuous Galerkin method for the diffusion term treated implicitly, and the regular discontinuous Galerkin method for the convection term treated explicitly, are unconditionally stable (the time step is upper-bounded only by a constant depending on the diffusion coefficient but not on the spatial mesh size) and optimally convergent. The results also hold for the drift-diffusion model in semiconductor device simulations, where a convection diffusion equation is coupled with an electrical potential equation. Numerical experiments confirm the good performance of such schemes. This is a joint work with Haijin Wang, Qiang Zhang and Yunxian Liu.

### Beatrice Riviere, Rice University

*High Order Methods for Flows in Heterogeneous Porous Media*

We formulate high order discontinuous Galerkin methods in space for coupled flow and transport problems in heterogeneous porous media. Heterogeneities include large variations in the permeability and porosity fields. We apply the method to two problems: a miscible displacement and a three-phase flow. Convergence of the method is obtained theoretically with minimum regularity assumption on the input data. Robustness and scalability of the method is obtained for three-dimensional problems.