

# A Theory for Wave Propagation in Porous Rocks Saturated by Two-Phase Fluids under Variable Pressure Conditions

BY C. L. RAVAZZOLI<sup>1</sup> AND J. E. SANTOS<sup>2</sup>

<sup>1</sup>*Facultad de Ciencias Astronómicas y Geofísicas, Universidad Nacional de La Plata, Paseo del Bosque S/N (1900) La Plata Argentina and CONICET Argentina. E-mail:claudia@fcaglp.fcaglp.unlp.edu.ar*

<sup>2</sup>*Facultad de Ciencias Astronómicas y Geofísicas, Universidad Nacional de La Plata, Paseo del Bosque S/N, (1900) La Plata Argentina; Dept. of Mathematics, Purdue University, 150 N. University Street West Lafayette, IN 47907-2068 USA and CONICET Argentina. E-mail:santos@fcaglp.fcaglp.unlp.edu.ar*

In this work we present an extension of Biot's theory to describe wave propagation in elastic and viscoelastic porous solids saturated by two-phase fluids under variable confining and pore pressure conditions. The model takes into account the capillary forces existing when the pore fluids are immiscible.

Appropriate bulk and pore volume compressibilities are defined in terms of the coefficients in the stress-strain relations, which lead to a generalization of the classic effective pressure laws for the case of single-phase fluids.

Using a Lagrangian formulation, the coupled equations of motion for the solid and the fluid phases are also derived, including dissipative effects due to matrix viscoelasticity and viscous coupling between the solid and fluid phases, which are used to model the levels of wave attenuation and dispersion observed in rocks.

Four different body waves can propagate in this type of media, three compressional waves and one shear wave. The sensitivity of the phase velocities and quality factors to variations in saturation and effective pressure in a sample of Boise sandstone saturated by a gas-water mixture is presented and analysed. Our results suggest that a combined analysis of such measurable quantities can help to determine the saturation and pressure states of a hydrocarbon reservoir.

**Keywords:** wave propagation, effective pressure, porous solids, immiscible fluids, rock physics.

## 1. INTRODUCTION

The analysis of the variation of seismic and mechanical properties of rocks when the saturant fluids are varied in their properties and composition is an active area of research in exploration geophysics, with application in reservoir characterization and engineering and also in petroleum geomechanics.

Theoretical formulations for the study of the deformation and elastic wave propagation in porous rocks with full, partial, multiphase, or segregate fluid saturation have been presented by different authors (see for example Biot 1956b,c; Biot 1962;

Berryman *et al.* 1988; Dutta & Odé 1979; Mochizuki 1982; Toksöz *et al.* 1976). However, none of these models incorporates the capillary forces existing when the pore fluids are immiscible. Consequently, the pressure variations induced by wave propagation in the different fluid phases are considered almost equal, neglecting possible changes in capillary pressure.

In this work we present a general theory for this kind of problems, which at the same time includes the effects of the ambient overburden pressure and the reference pressures of the immiscible fluids on the mechanical response of the rock. The theoretical basis were given by Santos *et al.* (1990a,b). In addition, generalizing the ideas given in Zimmerman *et al.* (1986) and in Zimmerman (1991) for single-phase fluids we define six compressibility coefficients to quantify the changes in either the pore volume or the bulk volume associated with changes in confining, wetting fluid and capillary pressures. Using these compressibilities, we established effective pressure laws for this model, showing that the effective pressure in this case depends not only on the wetting fluid pressure but also on the changes in capillary pressure.

For the study of wave propagation processes, two possible sources of energy dissipation are considered in this theory: Biot-type global flow and linear viscoelasticity. The first one is included by means of a viscous dissipation density function in the Lagrangian formulation and involves the relative flow velocities of the two fluids respect to the solid frame (Santos *et al.* 1990a). The second one is incorporated by extending the elastic constitutive relations to the linear viscoelastic case by means of a correspondence principle (see Biot 1956a). In this way the real poroelastic coefficients in the constitutive equations are replaced by complex frequency dependent poroviscoelastic moduli satisfying the same relations as in the elastic case. Viscoelastic behaviour is included in order to model the levels of dispersion and attenuation suffered by the different types of waves when travelling in real rocks. A form of the frequency correction factors for the mass and viscous coupling coefficients in the equations of motion needed in the high-frequency range is also presented. Moreover, generalizing a procedure previously applied to viscoelastic solids by Fabrizio & Morro (1992) and also to porous media saturated by single-phase fluids by Ravazzoli (1995), in the Appendix we analyse the resultant moduli using the laws of continuum equilibrium thermodynamics, leading us to establish some necessary restrictions. Finally, the theory is applied to predict and analyse the influence of the saturation, pore fluid types, capillarity and effective pressures on the phase velocities and quality factors in a sample of Boise sandstone.

## 2. A modified two-phase Biot model

Here we review the model presented by Santos *et al.* (1990a,b) describing the deformation and propagation of waves in an elastic isotropic, homogeneous, porous solid saturated by two immiscible fluids such as oil and water or gas and water. The original theory is modified to include explicitly in the formulation the absolute reference pressure of both fluid phases. Next, we derive the stress-strain relations, defining a set of elastic moduli and compressibilities for this kind of media.

## (a) Derivation of stress-strain relations

When two immiscible fluids occupy the voids of a porous solid one of them (depending on their adhesion tension), tends to preferentially *wet* the solid surface, spreading over it. In this way we can distinguish a *wetting* phase and a *non-wetting* one, which will be denoted with the subscripts (or superscripts) “*w*” and “*n*”, respectively. Let  $S_n$  and  $S_w$  denote the averaged wetting and non-wetting fluid saturations, respectively. Further, let us assume that the two fluid phases completely saturate the porous part of the bulk material so that  $S_n + S_w = 1$ .

The symbols  $S_{rw}$  and  $S_{rn}$  will denote the residual wetting and non-wetting fluid saturations, respectively. We assume that both fluids are allowed to move inside the pore space, so that  $S_{rn} < S_n < 1 - S_{rw}$  (see Bear 1972; Collins 1961; Peaceman 1977; Scheidegger 1974).

Let  $u^s$ ,  $\tilde{u}^n$ , and  $\tilde{u}^w$  denote the averaged absolute displacement vectors for the solid, non-wetting and wetting phases, respectively. Let  $\phi$  denote the effective porosity and for  $l = n, w$  set  $u^l = \phi(\tilde{u}^l - u^s)$ ,  $\xi^l = -\nabla \cdot u^l$ .

At a reference initial state, we consider a volume  $\bar{V}_b$  of homogeneous bulk material containing fluid volumes  $\bar{V}_n$  and  $\bar{V}_w$  at pressures  $\bar{P}_n$  and  $\bar{P}_w$ . In such a state we have  $\bar{V}_p = \bar{V}_n + \bar{V}_w$ ,  $\bar{S}_l = \bar{V}_l/\bar{V}_p$ ,  $l = n, w$ ,  $\bar{\phi} = \bar{V}_p/\bar{V}_b$ .

Note that for uniform porosity  $\bar{S}_n \xi^n$  and  $\bar{S}_w \xi^w$  represent the change in the corresponding fluid contents per unit volume of bulk material (Santos *et al.* 1990a). Thus, if  $\Delta V_l^c$  denotes the part of the total change in volume  $\Delta V_l = V_l - \bar{V}_l$  due to changes  $\Delta P_l = P_l - \bar{P}_l$  in the corresponding fluid pressures, since in equilibrium  $\nabla S_l = 0$ , we see that

$$\bar{S}_l \xi^l = (\Delta V_l - \Delta V_l^c) / \bar{V}_b = \bar{\phi} (\Delta V_l - \Delta V_l^c) / \bar{V}_p, \quad l = n, w. \quad (2.1)$$

Let  $K_n = C_n^{-1}$  and  $K_w = C_w^{-1}$  denote the bulk moduli of the non-wetting and wetting fluids, respectively,  $C_n$  and  $C_w$  being the corresponding compressibilities. Then, by definition

$$\Delta V_l^c / \bar{V}_l = -\Delta P_l / K_l, \quad l = n, w. \quad (2.2)$$

Also, neglecting second order terms,  $\Delta V_l = \bar{V}_p \Delta S_l + \bar{S}_l \Delta V_p$ ,  $l = n, w$ , and consequently

$$\xi^l = \bar{\phi} (\Delta S_l / \bar{S}_l + \Delta V_p / \bar{V}_p - \Delta V_l^c / \bar{V}_l), \quad l = n, w. \quad (2.3)$$

Setting  $\xi^* = \bar{S}_n \xi^n + \bar{S}_w \xi^w$ , it follows from (2.1) and (2.3) that

$$\xi^* = \bar{\phi} (\Delta V_p - \Delta V_n^c - \Delta V_w^c) / \bar{V}_p, \quad (2.4)$$

Next, let  $\tau_{ij} = \bar{\tau}_{ij} + \Delta \tau_{ij}$ ,  $i, j = 1, 2, 3$ , be the total stress tensor of the bulk material,  $\Delta \tau_{ij}$  being the change in the total stress with respect to a reference value  $\bar{\tau}_{ij}$  corresponding to the initial equilibrium state. In the same way,  $P_n = \bar{P}_n + \Delta P_n$  and  $P_w = \bar{P}_w + \Delta P_w$  denote the absolute pressures of the fluid phases,  $\Delta P_n$  and  $\Delta P_w$  representing small increments in the corresponding pressures with respect to their reference values  $\bar{P}_n$  and  $\bar{P}_w$ , and set  $\Delta S_n = S_n - \bar{S}_n = -\Delta S_w$ . Recall that  $P_n$  and  $P_w$  are related through the capillary relation (see Bear 1972; Peaceman 1977; Scheidegger 1974), which can be regarded as an equation of state for this system:

$$P_{ca}(S_n) = P_n - P_w = P_{ca}(\bar{S}_n) + \Delta P_{ca} \geq 0. \quad (2.5)$$

Then, ignoring terms of the second order in  $\Delta S_n$ ,

$$\Delta P_{ca}(\bar{S}_n + \Delta S_n) \cong P'_{ca}(\bar{S}_n)\Delta S_n. \quad (2.6)$$

The function  $P_{ca}(S_n)$  is a positive and strictly increasing function of the variable  $S_n$ . Hysteresis effects are ignored.

Let us consider a volume  $\Omega$  of bulk material of boundary  $\partial\Omega$  in static equilibrium state under the action of the surface forces  $f_i^s, f_i^n, f_i^w, i = 1, 2, 3$ , acting on the solid and fluid parts of  $\partial\Omega$  per unit area of bulk material. These forces can be written in the form (Santos *et al.* 1990a)

$$f_i^s = [\tau_{ij} + \phi(S_n P_n + S_w P_w)\delta_{ij}] \nu_j, \quad f_i^l = -\phi S_l P_l \delta_{ij} \nu_j, \quad l = n, w, \quad (2.7)$$

where  $\delta_{ij}$  denotes the Kronecker symbol and  $\nu_j$  is the unit normal to  $\partial\Omega$ .

If  $W^* = W^*(\Delta\tau_{ij}, \Delta P_n, \Delta P_w)$  represents the complementary strain energy density of the system, then its complementary potential energy  $\mathcal{V}^*$  is given by (see Santos *et al.* 1990a; Fung, 1965)

$$\mathcal{V}^* = \int_{\Omega} W^* dx - \int_{\partial\Omega} (f_i^s u_i^s + f_i^n \tilde{u}_i^n + f_i^w \tilde{u}_i^w) d\sigma, \quad (2.8)$$

where  $dx$  denotes an infinitesimal volume and  $d\sigma$  a surface element in the boundary  $\partial\Omega$ . Here and in what follows summation convention is used, i.e., sum on repeated indices is applied.

The principle of complementary virtual work for our system can be stated in the form (Santos *et al.* 1990a):

$$\begin{aligned} \int_{\Omega} \delta W^* dx &= \int_{\partial\Omega} (\delta f_i^s u_i^s + \delta f_i^n u_i^n + \delta f_i^w u_i^w) d\sigma \\ &\quad - \int_{\Omega} [\delta\lambda(\Delta P_n - \Delta P_w - \Delta P_{ca}) + \lambda(\delta\Delta P_n - \delta\Delta P_w - \delta\Delta P_{ca})] dx, \end{aligned} \quad (2.9)$$

where  $\delta$  denotes virtual changes and  $\lambda$  is a Lagrange multiplier used to introduce the capillary relation as a constraint. Neglecting second order terms in the different increments and using (2.6), it can be shown that

$$\begin{aligned} \delta f_i^n &= -\bar{\phi} \left[ \bar{S}_n \delta\Delta P_n + \bar{P}_n \frac{\delta\Delta P_{ca}}{P'_{ca}(\bar{S}_n)} \right] \delta_{ij} \nu_j, \quad \delta f_i^w = -\bar{\phi} \left[ \bar{S}_w \delta\Delta P_w - \bar{P}_w \frac{\delta\Delta P_{ca}}{P'_{ca}(\bar{S}_n)} \right] \delta_{ij} \nu_j, \\ \delta f_i^s &= -\bar{\phi} \left[ \bar{S}_n \delta\Delta P_n + \bar{S}_w \delta\Delta P_w + P_{ca}(\bar{S}_n) \frac{\delta\Delta P_{ca}}{P'_{ca}(\bar{S}_n)} \right] \delta_{ij} \nu_j + \delta\Delta\tau_{ij} \nu_j. \end{aligned} \quad (2.10)$$

Transforming the surface integral in (2.9) into a volume integral by means of Gauss theorem, using equations (2.10), and following the argument given by Santos *et al.* (1990a) it can be shown that

$$\begin{aligned} \delta W^* &= \varepsilon_{ij} \delta\Delta\tau_{ij} + (\bar{S}_n \xi^n - \lambda) \delta\Delta P_n + (\bar{S}_w \xi^w + \lambda) \delta\Delta P_w + \\ &\quad + (\beta \xi^n + \lambda + (\xi^n - \xi^w)\zeta) \delta\Delta P_{ca}, \end{aligned} \quad (2.11)$$

where  $\beta = P_{ca}(\bar{S}_n)/P'_{ca}(\bar{S}_n)$ ,  $\zeta = \bar{P}_w/P'_{ca}(\bar{S}_n)$ . Assuming that  $\delta W^*$  is an exact differential of the variables  $\Delta\tau_{ij}, \Delta P_n, \Delta P_w$ , and  $\Delta P_{ca}$ , the complementary strain

energy  $W^*$  is equal to the strain energy  $W$ , and using the capillary relation (2.5) in (2.11) we obtain

$$\begin{aligned} W &= \frac{1}{2} \{ \varepsilon_{ij} \Delta \tau_{ij} + \xi^n [(\bar{S}_n + \beta + \zeta) \delta \Delta P_n - (\beta + \zeta) \Delta P_w] \\ &\quad + \xi^w [(\bar{S}_w + \zeta) \Delta P_w - \zeta \Delta P_n] \} \\ &= \frac{1}{2} \{ \Delta \tau_{ij} \varepsilon_{ij} + [(\bar{S}_n + \beta + \zeta) \xi^n - \zeta \xi^w] \Delta P_n \\ &\quad + [(\bar{S}_w + \zeta) \xi^w - (\beta + \zeta) \xi^n] \Delta P_w \}. \end{aligned} \quad (2.12)$$

Restricting the analysis to linear stress–strain relations, for the isotropic elastic case we obtain the following stress–strain relations:

$$\begin{aligned} \text{i)} \quad & \Delta \tau_{ij} = 2N \varepsilon_{ij} + \delta_{ij} (\lambda_c e_b - B_1 \xi^n - B_2 \xi^w), \\ \text{ii)} \quad & \Delta \mathcal{T}_n = (\bar{S}_n + \beta + \zeta) \Delta P_n - (\beta + \zeta) \Delta P_w = -B_1 e_b + M_1 \xi^n + M_3 \xi^w, \\ \text{iii)} \quad & \Delta \mathcal{T}_w = (\bar{S}_w + \zeta) \Delta P_w - \zeta \Delta P_n = -B_2 e_b + M_3 \xi^n + M_2 \xi^w, \end{aligned} \quad (2.13)$$

where the magnitudes  $\Delta \tau_{ij}$ ,  $\Delta \mathcal{T}_n$  and  $\Delta \mathcal{T}_w$  represent the generalized forces for our system,  $\varepsilon_{ij}$  denotes the strain tensor, and  $e_b = \varepsilon_{ii} = \Delta V_b / \bar{V}_b$ . According to Gassmann (1951), it will be assumed that  $N$  is identical to the shear modulus of the dry rock  $N_m$ . Also,  $\lambda_c = K_c - \frac{2}{3}N$  in 3D and  $\lambda_c = K_c - N$  in 2D, with  $K_c = C_c^{-1}$  being the undrained bulk modulus, computed as in Santos *et al.* (1990b):

$$\begin{aligned} K_c &= K_s (K_m + \Xi) / (K_s + \Xi), \quad \Xi = K_f (K_m - K_s) / (\bar{\phi} (K_f - K_s)), \quad (2.14) \\ K_f &= \alpha (\gamma \bar{S}_n C_n + \bar{S}_w C_w)^{-1}, \quad \alpha = 1 + (\bar{S}_n + \beta)(\gamma - 1), \\ \gamma &= (1 + P'_{ca}(\bar{S}_n) \bar{S}_n \bar{S}_w C_w) (1 + P'_{ca}(\bar{S}_n) \bar{S}_n \bar{S}_w C_n)^{-1}, \end{aligned}$$

where  $K_m = C_m^{-1}$  and  $K_s = C_s^{-1}$  denote the bulk modulus of the empty matrix and the solid grains, respectively. In the equations above,  $K_f = C_f^{-1}$  defines an effective bulk modulus for a two–phase fluid taking into account capillary forces. Here we are assuming that the solid matrix is composed of a homogeneous material. In the case of mixed mineralogy an effective average coefficient for  $K_s$  can be used (see Mavko *et al.* 1998). The remaining coefficients can be obtained by using the following relations:

$$\begin{aligned} B_1 &= \chi K_c [(S_n + \beta)\gamma - \beta + (\gamma - 1)\zeta], \quad B_2 = \chi K_c [S_w + (1 - \gamma)\zeta], \quad (2.15) \\ M_1 &= -M_3 - B_1 C_m \eta^{-1}, \quad M_2 = (r B_2 + \zeta) q^{-1}, \quad M_3 = -M_2 - B_2 C_m \eta^{-1}, \end{aligned}$$

where

$$\begin{aligned} \chi &= [\eta + \bar{\phi} (C_m - C_c)] \{ \alpha [\eta + \bar{\phi} (C_m - C_f)] \}^{-1}, \quad q = \bar{\phi} (C_n + 1/P'_{ca}(\bar{S}_n) \bar{S}_n \bar{S}_w), \\ r &= (\bar{S}_n + \beta) C_s + (C_c - C_m) [q B_2 + (\bar{S}_n + \beta) (1 - C_s C_c^{-1})], \quad \eta = C_s - C_m, \end{aligned}$$

The inverse relations of (2.13) giving the generalized strains  $\varepsilon_{ij}$ ,  $(\bar{S}_n + \beta + \zeta) \xi^n - \zeta \xi^w$ , and  $(\bar{S}_w + \zeta) \xi^w - (\beta + \zeta) \xi^n$  as linear functions of  $\Delta \tau_{ij}$ ,  $\Delta P_n$ , and  $\Delta P_w$  are:

$$\begin{aligned} \text{i)} \quad & \varepsilon_{ij} = 1/(2N) \Delta \tau_{ij} + \delta_{ij} (D \Delta \tau - F_1 \Delta P_n - F_2 \Delta P_w), \\ \text{ii)} \quad & (\bar{S}_n + \beta + \zeta) \xi^n - \zeta \xi^w = -F_1 \Delta \tau + H_1 \Delta P_n + H_3 \Delta P_w, \quad (2.16) \\ \text{iii)} \quad & (\bar{S}_w + \zeta) \xi^w - (\beta + \zeta) \xi^n = -F_2 \Delta \tau + H_3 \Delta P_n + H_2 \Delta P_w. \end{aligned}$$

In the equations above  $\Delta \tau = \Delta \tau_{ii}$  and  $D, F_1, F_2, H_1, H_2, H_3$  are elastic coefficients (Santos *et al.* 1990a).

## (b) Definition of Bulk and Pore Compressibilities

Here we show that the model presented above enables us to establish a theory for the compressibilities of the system. For the analysis that follows we consider that the variable  $P_c$  (applied external pressure) plays the role of the *confining* or *overburden* pressure supported by the porous saturated sample at a given (fixed) depth.

First we proceed to derive expressions for  $e_b$  and  $e_p = \Delta V_p / \bar{V}_p$  that will be used to define the compressibilities for the saturated material. Consider tensional changes  $\Delta\tau_{ij}$  such that  $\Delta\tau_{ij} = 0$  for  $i \neq j$  and

$$\Delta\tau_{11} = \Delta\tau_{22} = \Delta\tau_{33} = \Delta\tau/3 = -\Delta P_c, \quad \Delta P_c > 0, \quad (2.17)$$

Note that from (2.2), (2.4) and (2.16i) we can express  $e_b$  and  $e_p$  in the form:

$$e_b = (3D + 1/2N)\Delta\tau - 3F_1\Delta P_n - 3F_2\Delta P_w \quad (2.18)$$

$$e_p = \xi^*/\bar{\phi} + \Delta V_p^c / \bar{V}_p = \xi^*/\bar{\phi} - \bar{S}_n C_n \Delta P_n - \bar{S}_w C_w \Delta P_w. \quad (2.19)$$

Next, adding (2.16ii) and (2.16iii) we obtain

$$\xi^* = -(F_1 + F_2)\Delta\tau + (H_1 + H_3)\Delta P_n + (H_2 + H_3)\Delta P_w. \quad (2.20)$$

Using the relations  $(3D + 1/2N) = C_m/3$ ,  $F_1 = (\bar{S}_n + \beta)\eta/3$ ,  $F_2 = (\bar{S}_w - \beta)\eta/3$  (see Santos *et al.* 1990b, pp. 1432) and combining (2.17), (2.20) and the capillary relation  $\Delta P_n - \Delta P_w = \Delta P_{ca}$ , equations (2.18) and (2.19) become

$$e_b = -C_m \Delta P_c - \eta \Delta P_w - (\bar{S}_n + \beta)\eta \Delta P_{ca}, \quad (2.21)$$

$$e_p = \eta/\bar{\phi} \Delta P_c + ([H_1 + H_2 + 2H_3]/\bar{\phi} - \bar{S}_n C_n - \bar{S}_w C_w) \Delta P_w + \quad (2.22)$$

$$+ ([H_1 + H_3]/\bar{\phi} - \bar{S}_n C_n) \Delta P_{ca}.$$

Since there are two volumes  $V_b$  and  $V_p$  and three different pressures ( $P_c$ ,  $P_w$ ,  $P_{ca}$ ) that may vary independently, from (2.21)–(2.22) we see that a set of six compressibilities can be defined as:

$$C_{bc} = -\frac{1}{\bar{V}_b} \left( \frac{\Delta V_b}{\Delta P_c} \right)_{P_w, P_{ca}} = C_m, \quad (2.23)$$

$$C_{bw} = \frac{1}{\bar{V}_b} \left( \frac{\Delta V_b}{\Delta P_w} \right)_{P_c, P_{ca}} = C_m - C_s, \quad (2.24)$$

$$C_{bca} = \frac{1}{\bar{V}_b} \left( \frac{\Delta V_b}{\Delta P_{ca}} \right)_{P_c, P_w} = (\bar{S}_n + \beta)(C_m - C_s), \quad (2.25)$$

$$C_{pc} = -\frac{1}{\bar{V}_p} \left( \frac{\Delta V_p}{\Delta P_c} \right)_{P_w, P_{ca}} = \frac{1}{\bar{\phi}}(C_m - C_s), \quad (2.26)$$

$$C_{pw} = \frac{1}{\bar{V}_p} \left( \frac{\Delta V_p}{\Delta P_w} \right)_{P_c, P_{ca}} = [H_1 + H_2 + 2H_3]/\bar{\phi} - \bar{S}_n C_n - \bar{S}_w C_w, \quad (2.27)$$

$$C_{pca} = \frac{1}{\bar{V}_p} \left( \frac{\Delta V_p}{\Delta P_{ca}} \right)_{P_c, P_w} = [H_1 + H_3]/\bar{\phi} - \bar{S}_n C_n. \quad (2.28)$$

Then, (2.21)–(2.22) can be stated in differential form as follows:

$$e_b = -C_{bc}(P_c, P_w, P_{ca})dP_c + C_{bw}(P_c, P_w, P_{ca})dP_w + C_{bca}(P_c, P_w, P_{ca})dP_{ca} \quad (2.29)$$

$$e_p = -C_{pc}(P_c, P_w, P_{ca})dP_c + C_{pw}(P_c, P_w, P_{ca})dP_w + C_{pca}(P_c, P_w, P_{ca})dP_{ca} \quad (2.30)$$

The coefficients  $C_{bc}$ ,  $C_{bw}$  and  $C_{pc}$  are similar to those defined in Zimmerman *et al.* (1986) and Zimmerman (1991) for the case of single-phase fluids with analogous interpretation and applications. The remaining coefficients  $C_{bca}$ ,  $C_{pw}$  and  $C_{pca}$  are strongly dependent on the saturation state and capillary pressure. This means that the changes in such variables can also modify the pore and bulk volumes.

Next note that from (2.23)–(2.24) and (2.26) we deduce that

$$\text{i) } C_{bw} = C_{bc} - C_s, \quad \text{ii) } C_{bw} = \bar{\phi}C_{pc}. \quad (2.31)$$

Equations (2.31i,ii) correspond to equations (2.5) and (2.6) in Zimmerman (1991). This shows the equivalence of the variable  $P_w$  and the “pore pressure”  $P_p$  used in such references and that  $C_{bw}$  plays the role of the compressibility  $C_{bp}$  in Zimmerman *et al.* (1986) and in Zimmerman (1991).

### 3. Effective pressure law for elastic bulk volume deformations

The infinitesimal pore and bulk strains found in the preceding Subsection correspond to pressure changes small enough so that the different compressibilities involved can be considered constant during the process. That analysis would be appropriate for elastic wave propagation problems. However, for other physical problems, the total integrated strain is required (Zimmerman 1991). Examples of such situations are the studies of subsidence and formation compaction due to pore fluid migration from hydrocarbon or groundwater reservoirs, the evaluation of the increase (decrease) in the pore volume of a rock due to a finite increase (decrease) in the pressure of the pore fluids and the estimation of the true in-situ porosity of a formation from laboratory measurements (usually made at zero confining pressure).

It is a well established fact that most of the mechanical and transport properties of cracked or porous media subjected to both external and internal pressures depend on *effective pressure*. This is a very important concept since the forementioned properties remain the same at a constant effective pressure, even though the confining and pore fluid pressures are varied. Important contributions to the formulation and analysis of effective stress rules, from both the theoretical and experimental points of view, were presented by different authors, as for example Terzaghi (1936), Geertsma (1957), Nur & Byerlee (1971), Christensen & Wang (1985), Zimmerman (1991), Gangi & Carlson (1996), Berryman (1992) and Prasad & Manghnani (1997).

Next we will show that the equations obtained in §2 *b* lead us to an extension of the effective pressure coefficients obtained by Zimmerman (1991) for uniform pore pressure systems, when capillary forces are considered. Assuming that  $e_b$  and  $e_p$  are exact differentials of the variables  $P_c$ ,  $P_w$  and  $P_{ca}$  and that  $C_s$  is constant, from (2.29) and (2.24) we get

$$\frac{\partial}{\partial P_w} \left[ \frac{\partial e_b}{\partial P_c} \right] = -\frac{\partial C_{bc}}{\partial P_w}, \quad (3.1)$$

$$\frac{\partial}{\partial P_c} \left[ \frac{\partial e_b}{\partial P_w} \right] = \frac{\partial C_{bw}}{\partial P_c} = \frac{\partial [C_{bc} - C_s]}{\partial P_c} = \frac{\partial C_{bc}}{\partial P_c}. \quad (3.2)$$

Consequently  $C_{bc} = C_{bc}(P_c - P_w, P_{ca})$ , and from (2.25) we also have that

$$C_{bca} = C_{bca}(P_c - P_w, P_{ca}) = (\bar{S}_n + \beta)(C_{bc}(P_c - P_w, P_{ca}) - C_s). \quad (3.3)$$

Next, following Zimmerman (1991), to obtain the total strain through any given process we integrate relations (2.29) and (2.30). To perform the integration in the  $(P_c, P_w, P_{ca})$  state variables, we move along the following paths:

$$\begin{aligned} \text{Path 1 : } & (0, 0, P_{ca}(S_n^*)) \rightarrow (P_c, 0, P_{ca}(S_n^*)) \\ \text{Path 2 : } & (P_c, 0, P_{ca}(S_n^*)) \rightarrow (P_c, \bar{P}_w, P_{ca}(S_n^*)) \\ \text{Path 3 : } & (P_c, \bar{P}_w, P_{ca}(S_n^*)) \rightarrow (P_c, \bar{P}_w, P_{ca}(\bar{S}_n)). \end{aligned}$$

The symbol  $S_n^*$  denotes an unstrained reference saturation state within the range  $S_{rn} < S_n^* < 1 - S_{rw}$ .

For the Path 1, the sample is subjected to an external pressure  $P_c$  and the wetting fluid pressure is held at  $P_w = 0$ , while non-wetting fluid is held at the reference pressure  $\bar{P}_n = P_{ca}(S_n^*)$  so that both pressures are constant during the experiment. Then from (2.29) we have that

$$E_b^1 = - \int_0^{P_c} C_{bc}(P_1 - 0, P_{ca}(S_n^*)) dP_1.$$

Next, for the Path 2, the external pressure  $P_c$  is held constant while both fluids receive an equal increment in pressure from 0 to  $\bar{P}_w$ . In this way there is no change in capillary pressure. The resulting bulk deformation during this process is

$$E_b^2 = \int_0^{\bar{P}_w} [C_{bc}(P_c - P_2, P_{ca}(S_n^*)) - C_s] dP_2 = - \int_{P_c}^{P_c - \bar{P}_w} C_{bc}(P, P_{ca}(S_n^*)) dP - C_s \bar{P}_w.$$

Finally for the Path 3 experiment the wetting fluid pressure is held at the pressure  $\bar{P}_w$  while the non-wetting fluid pressure is incremented from  $P_{ca}(S_n^*)$  to  $P_{ca}(\bar{S}_n)$ . This results in an increment in capillary pressure with constant  $\bar{P}_w$  and  $P_c$  with bulk volume deformation:

$$E_b^3 = \int_{P_{ca}(S_n^*)}^{P_{ca}(\bar{S}_n)} C_{bca}(P_c - \bar{P}_w, P_3) dP_3.$$

The total strain  $E_b$  is then given by the sum:  $E_b = E_b^1 + E_b^2 + E_b^3$ .

Next following Zimmerman (1991) we proceed to define the "secant" compressibilities  $\bar{C}_{bc}$  and  $\bar{C}_{bca}$  to obtain an effective stress form of the total strain  $E_b$ . Set  $\mathcal{R}(P_{ca}) = P_{ca}(\bar{S}_n) - P_{ca}(S_n^*)$  and let

$$\begin{aligned} \bar{C}_{bc}(P_c, P_{ca}(\bar{S}_n)) &= \frac{1}{P_c} \int_0^{P_c} C_{bc}(P, P_{ca}(\bar{S}_n)) dP \\ \bar{C}_{bca}(P_c - \bar{P}_w, P_{ca}(S_n^*), P_{ca}(\bar{S}_n)) &= \frac{1}{\mathcal{R}(P_{ca})} \int_{P_{ca}(S_n^*)}^{P_{ca}(\bar{S}_n)} C_{bca}(P_c - \bar{P}_w, P_3) dP_3 \\ &= \frac{1}{\mathcal{R}(P_{ca})} \int_{S_n^*}^{\bar{S}_n} [s + \beta(s)] [C_{bc}(P_c - \bar{P}_w, P_{ca}(s)) - C_s] P'_{ca}(s) ds, \end{aligned}$$



where we have used (2.25) and the fact that  $P_3 = P_3(s)$  where the variable  $s$  takes values in the saturation range  $[S_n^*, \bar{S}_n]$  and  $P_3(s)$  is the capillary pressure function. Then we see that the elastic deformation  $E_b$  of the bulk volume can be written in terms of an *effective pressure*  $P_{ef}^b$  in the form

$$\begin{aligned} E_b &\simeq -\bar{C}_{bc}(P_c - \bar{P}_w, P_{ca}(S_n^*))(P_c - \bar{P}_w) - C_s \bar{P}_w & (3.4) \\ &+ \bar{C}_{bca}(P_c - \bar{P}_w, P_{ca}(S_n^*), P_{ca}(\bar{S}_n)) \mathcal{R}(P_{ca}) \\ &= -\bar{C}_{bc}(P_c - \bar{P}_w, P_{ca}(S_n^*)) P_{ef}^b, \end{aligned}$$

where

$$P_{ef}^b = P_c - \bar{n}_{b1} \bar{P}_w - \bar{n}_{b2} \mathcal{R}(P_{ca}), \quad (3.5)$$

$$\bar{n}_{b1} = 1 - C_s \left/ \bar{C}_{bc}(P_c - \bar{P}_w, P_{ca}(S_n^*)), \right. \quad (3.6)$$

$$\bar{n}_{b2} = \bar{C}_{bca}(P_c - \bar{P}_w, P_{ca}(S_n^*), P_{ca}(\bar{S}_n)) \left/ \bar{C}_{bc}(P_c - \bar{P}_w, P_{ca}(S_n^*)). \right. \quad (3.7)$$

It is therefore seen that, the effect of the confining pressure is not only counteracted by the wetting fluid pressure (as may be expected) but also by a new term related to the capillary pressure change during the deformation process. A similar expression can be obtained for the total elastic deformation  $P_{ef}^p$  of the pore volume  $V_p$ . We do not include it here for brevity.

#### 4. The equations of motion

First we will get an expression for the complementary potential energy density  $\mathcal{V}_d^*$  of the system. Consider a perturbation of the system from the equilibrium state. According to (2.8)

$$\delta \mathcal{V}^* = \int_{\Omega} \delta \mathcal{V}_d^* dx = \int_{\Omega} \delta W^* dx - \int_{\partial \Omega} (u_i^s \delta f_i^s + \tilde{u}_i^n \delta f_i^n + \tilde{u}_i^w \delta f_i^w) d\sigma. \quad (4.1)$$

Using equations (2.10) and (2.11), we obtain

$$\delta \mathcal{V}^* = \int_{\Omega} \left\{ -u_i^s \delta \frac{\partial \Delta \tau_{ij}}{\partial x_j} + u_i^n \delta \frac{\partial}{\partial x_i} \Delta \mathcal{T}_n + u_i^w \delta \frac{\partial}{\partial x_i} \Delta \mathcal{T}_w + \lambda \delta [\Delta P_{ca} - (\Delta P_n - \Delta P_w)] \right\} dx.$$

As stated previously, since we are in the linear case, the complementary strain energy  $W^*$  equals the strain energy  $W$  and consequently the same holds for their corresponding densities  $\mathcal{V}_d^* = \mathcal{V}_d$ , (Santos *et al.* 1990a; Fung 1965). Then, assuming that  $\mathcal{V}_d$  is an exact differential we have

$$\mathcal{V}_d = -u_i^s \frac{\partial \Delta \tau_{ij}}{\partial x_j} + u_i^n \frac{\partial}{\partial x_i} \Delta \mathcal{T}_n + u_i^w \Delta \mathcal{T}_w + \lambda (\Delta P_c - (\Delta P_n - \Delta P_w)). \quad (4.2)$$

Thus, if  $u = (u_i^s, u_i^n, u_i^w) = (u_j)$ ,  $1 \leq i \leq 3$ ,  $1 \leq j \leq 9$ , are chosen as generalized coordinates and assuming that the system is conservative we get

$$\frac{\partial \mathcal{V}_d}{\partial u_i^s} = -\frac{\partial \Delta \tau_{ij}}{\partial x_j}, \quad \frac{\partial \mathcal{V}_d}{\partial u_i^n} = \frac{\partial}{\partial x_i} \Delta \mathcal{T}_n, \quad \frac{\partial \mathcal{V}_d}{\partial u_i^w} = \frac{\partial}{\partial x_i} \Delta \mathcal{T}_w, \quad 1 \leq i \leq 3. \quad (4.3)$$

Next, following Santos *et al.* (1990a), in the isotropic case the kinetic energy density  $E_c$  and the dissipation density function  $\mathcal{D}$  have the form

$$E_c = \frac{1}{2} \rho \frac{\partial u_i^s}{\partial t} \frac{\partial u_i^s}{\partial t} + \rho_n S_n \frac{\partial u_i^s}{\partial t} \frac{\partial u_i^n}{\partial t} + \rho_w S_w \frac{\partial u_i^s}{\partial t} \frac{\partial u_i^w}{\partial t} + \frac{1}{2} g_n^c \frac{\partial u_i^n}{\partial t} \frac{\partial u_i^n}{\partial t} + \frac{1}{2} g_w^c \frac{\partial u_i^w}{\partial t} \frac{\partial u_i^w}{\partial t} + g_{nw}^c \frac{\partial u_i^n}{\partial t} \frac{\partial u_i^w}{\partial t}, \quad (4.4)$$

$$\mathcal{D} = \frac{1}{2} (d_n^c \frac{\partial u_i^n}{\partial t} \frac{\partial u_i^n}{\partial t} + d_w^c \frac{\partial u_i^w}{\partial t} \frac{\partial u_i^w}{\partial t} - d_{nw}^c \frac{\partial u_i^n}{\partial t} \frac{\partial u_i^w}{\partial t}), \quad (4.5)$$

where  $\rho_w$  and  $\rho_n$  are the mass densities of the wetting and the non-wetting fluids and  $\rho$  is the density of the bulk material, i.e.,  $\rho = (1 - \bar{\phi})\rho_s + \bar{S}_n\rho_n + \bar{S}_w\rho_w$ , with  $\rho_s$  being the mass density of the solid grains. Also, the mass coupling coefficients  $g_n^c, g_w^c, g_{nw}^c$  represent the inertial effects associated with dynamic interactions among the three phases, while the coefficients  $d_n^c, d_w^c, d_{nw}^c$  include the viscous coupling between the solid and fluid phases. They can be computed by the formulae (see Santos *et al.* 1990a,b)

$$g_l^c = \mathcal{G} \rho_l \bar{S}_l / \bar{\phi}, \quad d_l^c = \mu_l (\bar{S}_l)^2 \mathcal{A}_l, \quad l = n, w, \quad (4.6)$$

$$g_{nw}^c = \epsilon \mathcal{G} (\rho_n \rho_w \bar{S}_n \bar{S}_w)^{\frac{1}{2}} / \bar{\phi}, \quad d_{nw}^c = (\mu_n \mu_w)^{\frac{1}{2}} \bar{S}_n \bar{S}_w K_{rnw} / \mathcal{A}, \quad (4.7)$$

The factor  $\mathcal{G}$  is known as a structure factor and is related to the tortuosity of the pore space; it can be estimated as follows (Berryman 1981):  $\mathcal{G} = \frac{1}{2} \left(1 + \frac{1}{\bar{\phi}}\right)$ . The constants  $\mu_n, \mu_w$  are the non-wetting and wetting fluid viscosities, respectively. Also,  $\mathcal{A} = K (K_{rn} K_{rw} - K_{rnw}^2)$ ,  $\mathcal{A}_n = K_{rw} / \mathcal{A}$ ,  $\mathcal{A}_w = K_{rn} / \mathcal{A}$ , with  $K, K_{rn}, K_{rw}$  and  $K_{rnw}$  denoting the absolute and relative permeabilities, respectively. The relative permeabilities are assumed to be functions of the non-wetting saturation  $S_n$  (Bear 1972; Peaceman 1977; Scheidegger 1974).

**Remark:** Note that (4.5) contains an additional viscous coupling coefficients  $d_{nw}^c$  not present in the original formulation in Santos *et al.* (1990a).

Finally, the Lagrange formulation of the equations of motion is given by

$$\frac{d}{dt} \left( \frac{\partial E_c}{\partial \dot{u}_j} \right) + \frac{\partial \mathcal{D}}{\partial \dot{u}_j} = - \frac{\partial \mathcal{V}_d}{\partial u_j}, \quad 1 \leq j \leq 9. \quad (4.8)$$

## 5. Inclusion of viscoelastic dissipation

So far, the only source of energy dissipation considered in the equations of motion is associated to the viscous solid-fluid interaction (global flow effect) given by (4.5), since the solid phase was assumed to be perfectly elastic. However, it is well known that wave dispersion and attenuation phenomena in real saturated rocks are higher than the associated to viscodynamic effects (Mochizuki 1982; Stoll & Bryan 1970; Carcione 2001). This is mainly due to the complexity of pore shapes, heterogeneities in the physical properties and in the distribution of the fluids and the intrinsic anelasticity of the frame. These factors can be included in the formulation by means of the theory of viscoelasticity. The theoretical basis for this generalization was given by Biot (1956a,1962), who developed the general theory of

deformation of porous materials saturated by viscous fluids when the solid phase exhibits linear viscoelastic behaviour. Using principles of irreversible thermodynamics he established a general operational relationship between generalized forces  $Q_i$  and observed coordinates  $q_i$ , of the form  $Q_i = T_{ij}q_j$ , where  $T_{ij}$  is a symmetric matrix. In this way Biot obtained a general correspondence rule between the elastic and viscoelastic formulations in the domain of the Laplace transform and showed that formally all the relations are identical. The poroviscoelastic formulation obtained in this way was later applied by different authors for the study of wave propagation problems (see Stoll 1974; Stoll & Bryan 1970; Keller 1989; Rasolofosaon 1991). It follows from (2.13) that the eight generalized forces of our model are related to the variables  $\xi^n, \xi^w$  and  $\varepsilon_{ij}$  by means of a symmetric matrix, whose elements are functions of the elastic coefficients. Thus, if we assume that the solid phase shows linear viscoelastic behaviour, we are able to extend the constitutive relations (2.13) by simply replacing the real elastic moduli  $N, \lambda_c, B_1, B_2, M_1, M_2, M_3$  by appropriate viscoelastic operators. Using Fourier transform in time (instead of Laplace), we can state this relations in the space–frequency domain as follows:

$$\begin{aligned} \text{i)} \Delta \widehat{\tau}_{ij}(\omega) &= 2\widehat{N}(\omega) \widehat{\varepsilon}_{ij}(\omega) + \delta_{ij}[\widehat{\lambda}_c(\omega) \widehat{e}_b(\omega) - \widehat{B}_1(\omega) \widehat{\xi}^n(\omega) - \widehat{B}_2(\omega) \widehat{\xi}^w(\omega)], \\ \text{ii)} \Delta \widehat{T}_n &= -\widehat{B}_1(\omega) \widehat{e}_b(\omega) + \widehat{M}_1(\omega) \widehat{\xi}^n(\omega) + \widehat{M}_3(\omega) \widehat{\xi}^w(\omega), \\ \text{iii)} \Delta \widehat{T}_w &= -\widehat{B}_2(\omega) \widehat{e}_b(\omega) + \widehat{M}_3(\omega) \widehat{\xi}^n(\omega) + \widehat{M}_2(\omega) \widehat{\xi}^w(\omega), \end{aligned} \quad (5.1)$$

where  $\omega = 2\pi f$  is the angular frequency,  $\widehat{N}, \widehat{\lambda}_c, \widehat{B}_1, \widehat{B}_2, \widehat{M}_1, \widehat{M}_2, \widehat{M}_3$  are complex frequency dependent *poroviscoelastic* moduli and the hat denotes time Fourier transform. A similar extension can be done for the strain-stress relations (2.16).

By definition, viscoelastic behaviour is characterized by the time dependent relationship between stresses and strains (*memory* effect). This can be clearly appreciated formulating (5.1) in the time domain. To obtain expressions analogous to the Boltzmann's integral formulation in viscoelasticity, first we need to introduce the *stress relaxation functions*  $R_J(t)$ ,  $J = N, \lambda_c, B_1, B_2, M_1, M_2, M_3$ , whose time derivatives  $\dot{R}_J(t)$  are related to the complex moduli in (5.1) as stated in (5.3) below. These relaxation functions characterize the response of the system when step-like strains are applied. In order to preserve the causality of the model it is necessary to assume that all the relaxation functions are zero for times  $t < 0$ , so that the stress at time  $t$  cannot be influenced by future strains. Taking this into account in the inverse Fourier integrals of eqns. (5.1) and integrating by parts assuming that  $\varepsilon_{ij}(-\infty) = \xi^n(-\infty) = \xi^w(-\infty) = 0$ , equations (5.1) can be stated in the space-time domain as follows:

$$\begin{aligned} \text{i)} \quad \Delta \tau_{ij}(t) &= 2R_N(0+)\varepsilon_{ij}(t) + \delta_{ij} [R_{\lambda_c}(0+)e_b(t) - R_{B_1}(0+)\xi^n(t) - R_{B_2}(0+)\xi^w(t)] \\ &+ 2 \int_0^\infty \dot{R}_N(s)\varepsilon_{ij}(t-s)ds + \delta_{ij} \int_0^\infty \left[ \dot{R}_{\lambda_c}(s)e_b(t-s) \right. \\ &\quad \left. - \dot{R}_{B_1}(s)\xi^n(t-s) - \dot{R}_{B_2}(s)\xi^w(t-s) \right] ds, \end{aligned} \quad (5.2)$$

$$\begin{aligned}
\text{ii)} \quad \Delta \mathcal{T}_n(t) &= -R_{B_1}(0+)e_b(t) + R_{M_1}(0+)\xi^n(t) + R_{M_3}(0+)\xi^w(t) \\
&+ \int_0^\infty \left[ -\dot{R}_{B_1}(s)e_b(t-s) + \dot{R}_{M_1}(s)\xi^n(t-s) + \dot{R}_{M_3}(s)\xi^w(t-s) \right] ds, \\
\text{iii)} \quad \Delta \mathcal{T}_w(t) &= -R_{B_2}(0+)e_b(t) + R_{M_3}(0+)\xi^n(t) + R_{M_2}(0+)\xi^w(t) \\
&+ \int_0^\infty \left[ -\dot{R}_{B_2}(s)e_b(t-s) + \dot{R}_{M_3}(s)\xi^n(t-s) + \dot{R}_{M_2}(s)\xi^w(t-s) \right] ds.
\end{aligned}$$

In general, the notation  $R_j(0+)$  denotes  $\lim_{t \rightarrow 0+} R_j(t)$ . Let  $\hat{F}^S(\omega)$  and  $\hat{F}^C(\omega)$  be the half-range Fourier sine and cosine transforms of the function  $\hat{f}(t)$ . Then, the following frequency-domain relations between complex moduli and relaxation functions can be shown:

$$\hat{J}(\omega) = \hat{J}_R(\omega) + i\hat{J}_I(\omega) = R_J(0+) + \dot{R}_J^C(\omega) - i\dot{R}_J^S(\omega), \quad (5.3)$$

where  $J$  stands for  $N, \lambda_c, B_1, B_2, M_1, M_2, M_3$ .

As pointed out in Zimmerman (1991), although the derivatives appearing in the different compressibilities (2.23)–(2.28) can be also defined for irreversible deformation processes, such coefficients are generally computed only for purely elastic cases. Then for consistency, the compressibilities of the model could be extended to the present case by considering the *relaxed limits* (i.e.  $\lim \omega \rightarrow 0$  or  $\lim t \rightarrow \infty$ ) of the complex moduli appearing in (5.3). In Appendix A we give a set of restrictions imposed by the Laws of Thermodynamics on the imaginary parts of the coefficients in (5.1).

## 6. The equations of motion for the full frequency range.

It is known that for single-phase fluids, in the high-frequency range the viscous and mass coupling coefficients become frequency dependent (see Biot 1956c; Johnston *et al.* 1987, Carcione 2001). This effect is associated with the departure of the flow from the laminar Poiseuille type at the pore scale, which occurs for frequencies greater than some characteristic value. Since we have two immiscible fluids flowing within the poral space and three relative permeability functions, following the ideas in Berryman *et al.* 1988, we define for this model three *characteristic frequencies* of the form

$$\omega_c^l = \bar{S}_l \mu_l \bar{\phi} \mathcal{A}_l / (\mathcal{G} \rho_l), \quad l = n, w, \quad (6.1)$$

$$\omega_c^{nw} = \bar{\phi} (\mu_n \mu_w)^{\frac{1}{2}} (\bar{S}_n \bar{S}_w)^{\frac{1}{2}} K_{rnw} / (\epsilon \mathcal{A} \mathcal{G} (\rho_n \rho_w)^{\frac{1}{2}}). \quad (6.2)$$

These frequencies are obtained by equating the inertial and viscous drag terms associated with each fluid phase and using (4.6) and (4.7). For frequencies bigger than the minimum of the three characteristic frequencies defined above, the mass and viscous coupling coefficients are defined as follows:

$$g_l(\omega) = g_l^c + \mu_l (\bar{S}_l)^2 \mathcal{A}_l F_I(\theta_l) / \omega, \quad l = n, w, \quad (6.3)$$

$$g_{nw}(\omega) = g_{nw}^c + (\mu_n \mu_w)^{\frac{1}{2}} \bar{S}_n \bar{S}_w K_{rnw} F_I(\theta_{nw}) / (\omega \mathcal{A}), \quad (6.4)$$

$$d_l(\omega) = d_l^c F_R(\theta_l), \quad l = n, w, nw \quad (6.5)$$

The complex valued frequency dependent function  $F(\theta_j) = F_R(\theta_j) + iF_I(\theta_j)$ ,  $j = n, w, nw$  in (6.3)–(6.5) is the “universal” frequency correction function defined by Biot (1956c) for single phase fluids:

$$F(\theta) = \frac{1}{4} \frac{\theta T(\theta)}{1 - \frac{2}{i\theta} T(\theta)}, \quad T(\theta) = \frac{\text{ber}'(\theta) + i\text{bei}'(\theta)}{\text{ber}(\theta) + i\text{bei}(\theta)},$$

with  $\text{ber}(\theta)$  and  $\text{bei}(\theta)$  being the Kelvin functions of the first kind and zero order. The arguments  $\theta_j$  for  $F(\theta_j)$ ,  $j = n, w, nw$  in (6.3)–(6.5) can be estimated as in Biot (1956c) and Santos *et al.* (1992):

$$\theta_j = a_p^j \sqrt{\omega \rho_j / \mu_j}, \quad a_p^j = 2\sqrt{K k_{rj} A_0 / \bar{\phi}}, \quad j = n, w, nw, \quad (6.6)$$

where  $A_0$  denotes the Kozeny-Carman constant (see Bear 1972; Hovem & Ingram 1979).

Next, combine (4.8) with (4.3), (4.4), (4.5) and (5.1) and assume that  $S_n$  is independent of time (see Santos *et al.* 1990a for the argument justifying this assumption). For a spatially homogeneous medium, the equations of motion (4.8) formulated in the space-frequency domain take the following form:

$$\begin{aligned} -\omega^2 (\rho \hat{u}^s + \rho_n \bar{S}_n \hat{u}^n + \rho_w \bar{S}_w \hat{u}^w) &= (\widehat{K}_c + \frac{4}{3} \widehat{N}) \nabla \nabla \cdot \hat{u}^s \\ &\quad - \widehat{N} \nabla \times \nabla \times \hat{u}^s + \widehat{B}_1 \nabla (\nabla \cdot \hat{u}^n) + \widehat{B}_2 \nabla (\nabla \cdot \hat{u}^w) \\ -\omega^2 (\rho_n \bar{S}_n \hat{u}^s + g_n \hat{u}^n + g_{nw} \hat{u}^w) + i\omega d_n \hat{u}^n - i\omega d_{nw} \hat{u}^w &= \widehat{B}_1 \nabla (\nabla \cdot \hat{u}^s) \\ &\quad + \widehat{M}_1 \nabla (\nabla \cdot \hat{u}^n) + \widehat{M}_3 \nabla (\nabla \cdot \hat{u}^w) \\ -\omega^2 (\rho_w \bar{S}_w \hat{u}^s + g_{nw} \hat{u}^n + g_w \hat{u}^w) + i\omega d_w \hat{u}^w - i\omega d_{nw} \hat{u}^n &= \widehat{B}_2 \nabla (\nabla \cdot \hat{u}^s) \\ &\quad + \widehat{M}_3 \nabla (\nabla \cdot \hat{u}^n) + \widehat{M}_2 \nabla (\nabla \cdot \hat{u}^w). \end{aligned} \quad (6.7)$$

Taking into account the fact that  $F_R(\theta) \rightarrow 1$  and  $F_I(\theta)/\omega \rightarrow 0$  as  $\omega \rightarrow 0$  we may regard (6.7) as the general form of the equations of motion for frequencies ranging from the seismic to the ultrasonic range.

The plane wave analysis performed in Santos *et al.* (1990a) shows that in these type of media, three different compressional waves (Type I, Type II and Type III) and one shear wave (or S-wave) can propagate. The Type-I wave is the analogue of the classical fast P-wave propagating in elastic or viscoelastic isotropic solids and is associated with the motion of both solid and fluids moving nearly in phase. The Type-II and Type-III are slow waves strongly attenuated in the low frequency range. The first one is analogous to the classical Biot’s slow wave, with the two fluids moving nearly in phase and in opposite phase with the solid. For the Type III waves, both fluids move in nearly opposite phase with each other. This is a *new mode* not present in the case of single-phase fluids.

Denoting by  $k^j$ ,  $j = I, II, III, S$ , the complex wavenumbers, the phase velocities  $C$  and quality factors  $Q$  are obtained by using the formulae:

$$C_j = \omega / \text{Re}(k_j), \quad Q_j = -\text{Re}(k_j) / 2\text{Im}(k_j), \quad (6.8)$$

	$P_0$	$y_0$	$A_1$	$t_1$	$A_2$	$t_2$
$(K_m)^{-1}$	0.536626645088	0.092089	0.01766	7.634	0.01828	38.53
$(N_m)^{-1}$	4.427347752235	0.10113	0.012	1.925	0.01259	23.71

Table 1. Coefficients of equation (7.1) for Boise sandstone. For pressures given in MPa the moduli result in GPa.

## 7. Application to a real sandstone

We use the model to analyse the combined effects of saturation and effective pressure on the attributes of the different waves propagating in a sample of Boise sandstone, a well consolidated feldspathic graywacke (see King 1966 and Mann & Fatt 1960). Its material properties are  $\bar{\phi}=0.25$ ,  $K=1400 \cdot 10^{-15} \text{ m}^2$ ,  $K_s=34.5 \text{ GPa}$  and  $\rho_s=2550 \text{ kg/m}^3$ . To introduce the variation of the matrix properties with effective pressure, we obtained the dependence of the shear and bulk dry-rock moduli  $N_m(P_c)$  and  $K_m(P_c)$  versus confining pressure by assuming that for each  $P_c$ , the measured velocities correspond to the elastic isotropic approximation. Then, we performed exponential regressions of the form:

$$f^{-1}(P_c) = y_0 + A_1 \exp(-(P_c - P_0)/t_1) + A_2 \exp(-(P_c - P_0)/t_2), \quad (7.1)$$

where  $f$  stands for  $N_m$  or  $K_m$ . The coefficients are given in Table I. Since for the dry sample the effective pressure equals the confining pressure, to obtain the elastic properties of the skeleton for a given combination of confining and fluid pressures, we simply replace  $P_c$  by the effective pressure  $P_{ef}^b$  (given by (3.5)) in equation (7.1). The sample is subjected to a fixed confining pressure of 60 MPa and the pore space is assumed to be filled with water (as the wetting phase) and a hydrocarbon gas. Their properties are:  $\rho_w=1000 \text{ kg/m}^3$ ,  $\mu_w=0.01 \text{ N s/m}^2$ ,  $K_w=2.223 \text{ GPa}$ ,  $\rho_n=100 \text{ kg/m}^3$ ,  $\mu_n=0.00015 \text{ N s/m}^2$ ,  $K_n=0.022 \text{ GPa}$ . For the computations the pressure of the water was varied within the range:  $25 \text{ MPa} \leq \bar{P}_w \leq 60 \text{ MPa}$ .

The capillary pressure function  $P_{ca}(S_n)$  and the relative permeability functions  $K_{rn}(S_n)$  and  $K_{rw}(S_n)$  needed to describe our system are taken to be (see Douglas *et al.* 1993):

$$P_{ca}(S_n) = A \left( 1/(S_n + S_{rw} - 1)^2 - S_{rn}^2/[S_n(1 - S_{rn} - S_{rw})]^2 \right), \quad (7.2)$$

$$K_{rn}(S_n) = (1 - (1 - S_n)/(1 - S_{rn}))^2, \quad K_{rw}(S_n) = ([1 - S_n - S_{rw}]/(1 - S_{rw}))^2.$$

These relations are based on laboratory experiments performed on different porous rocks during imbibition and drainage processes (neglecting hysteresis effects). We chose  $S_{rw} = S_{rn} = 0.05$ , and  $A = 30 \text{ kPa}$ . In the absence of proper experimental data, the coupling permeability function  $K_{rnw}(S_n)$  used in this work is assumed to be  $K_{rnw}(S_n) = \sqrt{\epsilon K_{rn}(S_n) K_{rw}(S_n)}$ . The parameter  $\epsilon$  in (4.7) and the definition of  $K_{rnw}$  is equal to 0.1, as in Santos *et al.* (1990b). The whole set of poroelastic moduli is computed using (2.14)-(2.15). The viscoelastic behaviour is introduced using the phenomenological model defined by Liu *et al.* (1976) to describe attenuation by making the shear and undrained modulus complex and frequency dependent, while all the other coefficients remain real. This is a linear and causal model, whose associated complex modulus behaves properly in all the frequency range, and satisfies

the restrictions imposed by the Laws of Thermodynamics (see Appendix A). Using this model, the complex moduli  $\widehat{K}_c = \widehat{K}_c(\omega, P_{ef}^b, \bar{S}_n)$ ,  $\widehat{N} = \widehat{N}(\omega, P_{ef}^b)$  are computed as

$$\widehat{K}_c = \frac{K_c^r(P_{ef}^b, \bar{S}_n)}{R_{K_c}(\omega, P_{ef}^b) - iT_{K_c}(\omega, P_{ef}^b)}, \quad \widehat{N} = \frac{N^r(P_{ef}^b)}{R_N(\omega, P_{ef}^b) - iT_N(\omega, P_{ef}^b)}. \quad (7.3)$$

The real coefficients  $K_c^r(P_{ef}^b)$  and  $N^r(P_{ef}^b)$  denote the relaxed closed bulk and shear moduli, respectively. They are chosen so that the high frequency limits of (7.3) match the values of  $N(P_{ef}^b)$  and  $K_c(P_{ef}^b, \bar{S}_n)$  obtained from (7.1) and (2.14), respectively. The frequency dependent functions  $R_l$  and  $T_l$ ,  $l = K_c, N$ , associated with a continuous spectrum of relaxation times, characterize the viscoelastic behaviour and are given by (see Liu *et al.* 1976, Carcione 2001)

$$R_l(\omega, P_{ef}^b) = 1 - \frac{1}{\pi \widehat{Q}_l(P_{ef}^b)} \ln \frac{1 + \omega^2 T_{1,l}^2}{1 + \omega^2 T_{2,l}^2}, \quad (7.4)$$

$$T_l(\omega, P_{ef}^b) = \frac{2}{\pi \widehat{Q}_l(P_{ef}^b)} \tan^{-1} \frac{\omega(T_{1,l} - T_{2,l})}{1 + \omega^2 T_{1,l} T_{2,l}}, \quad l = K_c, N. \quad (7.5)$$

The parameters in (7.4)-(7.5) are taken such that the resulting quality factors  $Q_l = T_l/R_l$  are approximately equal to the reference values  $\widehat{Q}_l$  in the range of frequencies where the model is applied. For the computations we take  $T_{1,l} = 10^1 0 \frac{1}{\text{ms}}$ ,  $T_{1,l} = 10^9 \frac{1}{\text{ms}}$ , for  $l = K_c, N$ . We also introduced the dependence of the quality factors  $\widehat{Q}_{K_c}$  and  $\widehat{Q}_N$  on effective pressure (Winkler & Nur, 1979), disregarding for simplicity any possible dependence of such coefficients on saturation. The value of the Kozeny-Carman constant  $A_0$  in (6.6) is equal to 5.

According to (6.1)-(6.2) the characteristic angular frequencies at 10 % gas saturation are about  $\omega_c^n = 430$  kHz,  $\omega_c^w = 89$  kHz and  $\omega_c^{nw} = 182$  kHz.

Figure 1 (a) shows the resulting bulk effective pressure  $P_{ef}^b$  versus wetting fluid pressure for 40 % gas saturation. We compare our results with other estimates, such as the wetting-differential pressure, defined as  $P_{wd} = P_c - \bar{P}_w$  and a general law proposed by Gangi & Carlson (1996) of the form:

$$P_{ef}^G = P_c - n_G \bar{P}_w, \quad \text{with } n_G = n_0 - n_1 \bar{P}_w, \quad n_0 = 1, \quad n_1 = 0.014 \text{ MPa}^{-1}.$$

Significant differences can be observed in almost all the pressure range. As expected, the increase in wetting fluid pressure causes a reduction in the effective pressure acting on the solid matrix, producing a ‘‘softening’’ effect in the sandstone.

Figure 1 (b) illustrates the coefficients  $n_{b1}$  and  $n_{b2}$  versus  $\bar{P}_w$  for different saturation states. We observe that when  $\bar{P}_w$  approaches  $P_c$  (i.e., near the fracture limit),  $n_{b1} \rightarrow 1$  and  $n_{b2} \rightarrow 0$ , and  $P_{ef}^b \rightarrow P_{wd}$  (since  $\bar{C}_{bc} \rightarrow \infty$  for  $\bar{P}_w \rightarrow P_c$ ). Unlike the ‘‘classic’’  $n_{b1}$  coefficient,  $n_{b2}$  is strongly dependent on saturation, showing the influence of the capillary forces on effective pressure, an effect that should be carefully investigated with laboratory measurements.

Next we analyse the behaviour of the phase velocities and quality factors for the different wave propagation modes in the ultrasonic frequency range (for  $f = 1$  MHz). In Figures 2 (a) and (b) we plot the phase velocities of the Type I compressional wave and the Shear wave versus  $P_{ef}^b$  for different saturation states. The marked

increase observed in both velocities with effective pressure reflects the change in the elastic properties of the skeleton with effective pressure given by (7.1). This effect is mainly associated to the closure of microcracks, low aspect ratio pores and loose grain contacts, which increase the stiffness of the rock. For low effective pressures both velocities show a significant decrease, an effect usually observed in formations with very high pore pressures (i.e. overpressured), and also predicted by different theories (see Carcione & Gangi 2000; Toksöz *et al.* 1976).

Figures 3 (a) and (b) show the behaviour of the Type II and Type III compressional waves. These “slow” propagation modes are clearly more sensitive to saturation also showing a strong dependence on effective pressure.

Finally, the quality factors for the Type I and Shear waves are shown in Figure 4 (a) and (b). As observed in the phase velocities, these coefficients show an important monotonic increase with effective pressure, in agreement with the observations made by different authors (see Carcione & Gangi 2000; Schön 1996; Tao *et al.* 1995). The quality factors associated to the slow waves are not included in the paper for brevity.

## 8. Conclusions

In this work we developed a theory to study the processes of deformation and wave propagation in porous solids saturated by two immiscible fluids. The model allows the inclusion of many parameters such as porosity, permeability, lithology, pore fluid types, saturation state, confining pressure, fluid pressures and the capillary pressure vs. saturation curve, which are very important in different fields such as rock physics, interpretation of laboratory data and reservoir characterization.

We derived the elastic stress-strain relations, introducing appropriate elastic moduli. Generalizing a classic approach we defined a set of compressibilities and established a new effective pressure law for elastic bulk volume deformations. Its behaviour is consistent with other estimates and models the well known effect of decreasing of seismic velocities observed in overpressured formations. The new coefficient  $n_{b2}$  is strongly dependent on saturation, showing the influence of the capillary forces on effective pressure. The procedure can be readily extended to analyse pore volume deformations.

The equations of motion, valid from the seismic to the ultrasonic range, were formulated in the space-frequency domain, taking into account attenuation and dispersion effects associated to frequency dependent viscodynamic effects and matrix viscoelasticity.

The model was applied to study the influence of saturation and variable fluid pressures on dilatational and shear wave velocities and the corresponding quality factors in a sample of Boise sandstone. The wave velocities are very sensitive to effective pressure and in particular, those of the Type II and Type III waves are strongly dependent on saturation state. This is also observed in the corresponding quality factors. This suggests that combined analysis of phase velocities and quality factors can be used as indicators of the saturation and pressure states of a reservoir rock.

The results relative to Type I and Shear waves are in good agreement with published experimental and theoretical works. Those corresponding to the slow waves should be checked in the laboratory and we hope this will motivate further experimental work in this subject.



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## Appendix A. Thermodynamic restrictions

Here we derive a set of restrictions imposed by the Laws of Thermodynamics on the imaginary parts of the coefficients in (5.1). It will be assumed that the principles of continuum thermodynamics are valid for our system at the macroscopic scale. A different approach was presented by De la Cruz *et al.* (1993), based on the energy balance equations in the solid and the fluid phases and using averaging techniques.

Combining the First and Second Laws of Thermodynamics and using a generalization of the argument given in Fabrizio & Morro (1992) for viscoelastic materials and in Ravazzoli (1995) for poroviscoelastic solids saturated by single-phase fluids, for any isothermic cycle of period  $2\pi/\omega$  the following inequality must hold:

$$\int_0^{2\pi/\omega} \left( \Delta\tau_{ij}\dot{\varepsilon}_{ij} + \Delta\mathcal{T}_n\dot{\xi}^n + \Delta\mathcal{T}_n\dot{\xi}^w \right) dt \geq 0. \quad (\text{A } 1)$$

Next, we choose  $\varepsilon_{ij}$ ,  $\xi^n$  and  $\xi^w$  in (A 1) and in the stress–strain relations (5.2) to be of the form  $\varepsilon_{ij} = A_{ij}(\cos\omega t + \sin\omega t)$ ,  $\xi^l = \xi^l(\cos\omega t + \sin\omega t)$ ,  $\omega \geq 0$ , where  $A_{ij}$  is a symmetric matrix. Set  $e_A = A_{ii}$  and note that  $e_b(t) \equiv \varepsilon_{ii} = e_A(\cos\omega t + \sin\omega t)$ . A lengthy calculation yields

$$\begin{aligned} & e_A^2 \dot{R}_{\lambda_c}^S(\omega) + 2\dot{R}_N^S(\omega)A_{ij}A_{ij} - 2\xi^n e_A \dot{R}_{B_1}^S(\omega) - 2\xi^w e_A \dot{R}_{B_2}^S(\omega) \\ & + (\xi^n)^2 \dot{R}_{M_1}^S(\omega) + (\xi^w)^2 \dot{R}_{M_2}^S(\omega) + 2\xi^n \xi^w \dot{R}_{M_3}^S(\omega) \leq 0, \quad \omega \geq 0, \end{aligned} \quad (\text{A } 2)$$

which is an extension of Graffi’s inequality for this type of media (see Fabrizio & Morro 1992). Setting  $\widehat{K}_{cI}(\omega) = \widehat{\lambda}_{cI}(\omega) + \frac{2}{3}\widehat{N}_I(\omega)$  and combining (5.3) and the identity

$$2A_{ij}A_{ij} = \frac{2}{3} [e_A^2 + (A_{11} - A_{22})^2 + (A_{11} - A_{33})^2 + (A_{22} - A_{33})^2] + 4(A_{12}^2 + A_{13}^2 + A_{23}^2),$$

we see that equation (A 2) can be stated in the form

$$\begin{aligned} & e_A^2 \widehat{K}_{cI}(\omega) - 2\xi^n e_A \widehat{B}_{1I}(\omega) - 2\xi^w e_A \widehat{B}_{2I}(\omega) + (\xi^n)^2 \widehat{M}_{1I}(\omega) + (\xi^w)^2 \widehat{M}_{2I}(\omega) \\ & + 2\xi^n \xi^w \widehat{M}_{3I}(\omega) + 4[(A_{12}^2 + A_{13}^2 + A_{23}^2) \\ & + \frac{2}{3} [(A_{11} - A_{22})^2 + (A_{11} - A_{33})^2 + (A_{22} - A_{33})^2]] \widehat{N}_I(\omega) \geq 0, \quad \omega \geq 0, \end{aligned} \quad (\text{A } 3)$$

Next we consider the case of a hydrostatic compression, i.e.,  $A_{ij} = 0$ ,  $i \neq j$  and  $A_{11} = A_{22} = A_{33}$ . From (A 3) we get

$$\vec{Z}^t \widehat{P}_I(\omega) \vec{Z} \geq 0, \quad \omega \geq 0, \quad (\text{A } 4)$$

where  $\vec{Z}^t = (e_A, \xi^n, \xi^w)$  and the symmetric matrix  $\widehat{P}_I$  is defined by  $\widehat{P}_{I11} = \widehat{K}_{uI}$ ,  $\widehat{P}_{I22} = \widehat{M}_{1I}$ ,  $\widehat{P}_{I33} = \widehat{M}_{2I}$ ,  $\widehat{P}_{I12} = -\widehat{B}_{1I}$ ,  $\widehat{P}_{I13} = -\widehat{B}_{2I}$ ,  $\widehat{P}_{I23} = \widehat{M}_{3I}$ .

Next for a pure shear oscillation, i.e,  $A_{ij} \neq 0, i \neq j, A_{11} = A_{22} = A_{33} = 0$  and  $\xi_1^n = \xi_1^w = 0$ , from (A 3) we have

$$\widehat{N}_I(\omega) \geq 0, \quad \omega \geq 0. \quad (\text{A } 5)$$

Inequalities (A 4) and (A 5) are the thermodynamic restrictions imposed on the imaginary parts of the complex frequency dependent coefficients in the stress-strain relations (5.1).

**Remark:** The equality in (A 4) and (A 5) holds if and only if  $\omega = 0$ , i.e., the system behaves as elastic at zero frequency. This results from (5.3) and the definitions of the half-range Fourier sine and cosine transforms.

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### Figure Captions

**Figure 1.** (a) Comparison between effective pressure, wetting differential pressure and Gangi-Carlson's type law for  $S_n = 0.4$ . (b) Behaviour of  $n_{b1}$  and  $n_{b2}$  coefficients versus  $P_w$  for different saturation states.

**Figure 2.** (a) - (b) Phase velocities of Type I - P and Shear waves vs.  $P_{ef}^b$  for different saturations.

**Figure 3.** (a) - (b) Phase velocities of Type II and III - P waves vs.  $P_{ef}^b$  for different saturations.

**Figure 4.** (a) - (b) Quality Factors of Type I - P and Shear waves vs.  $P_{ef}^b$  for different saturations.

Figure 1

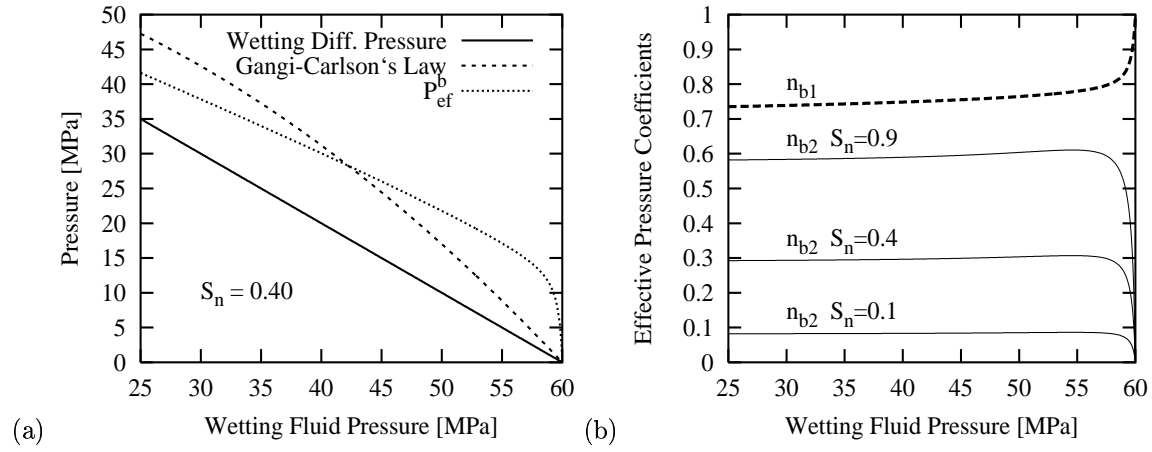


Figure 2

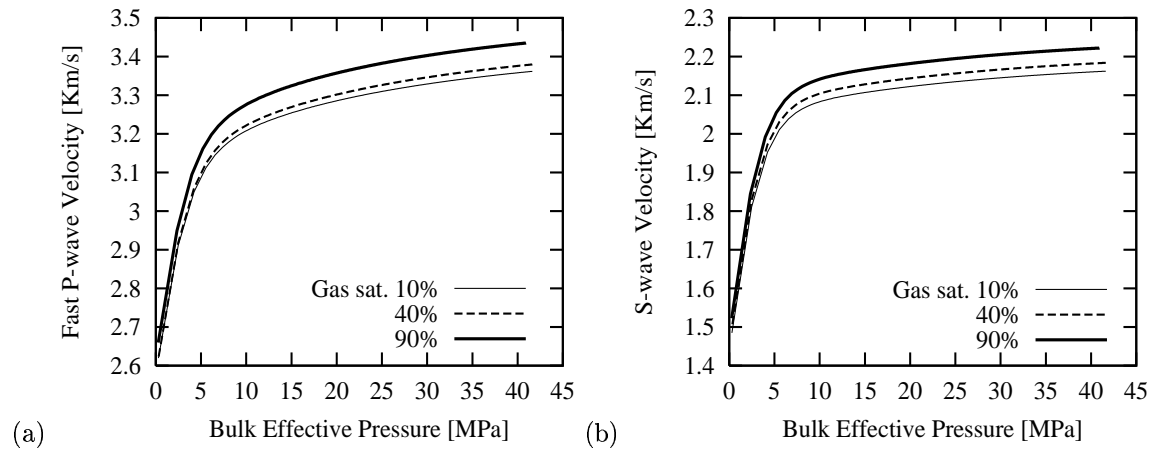


Figure 3

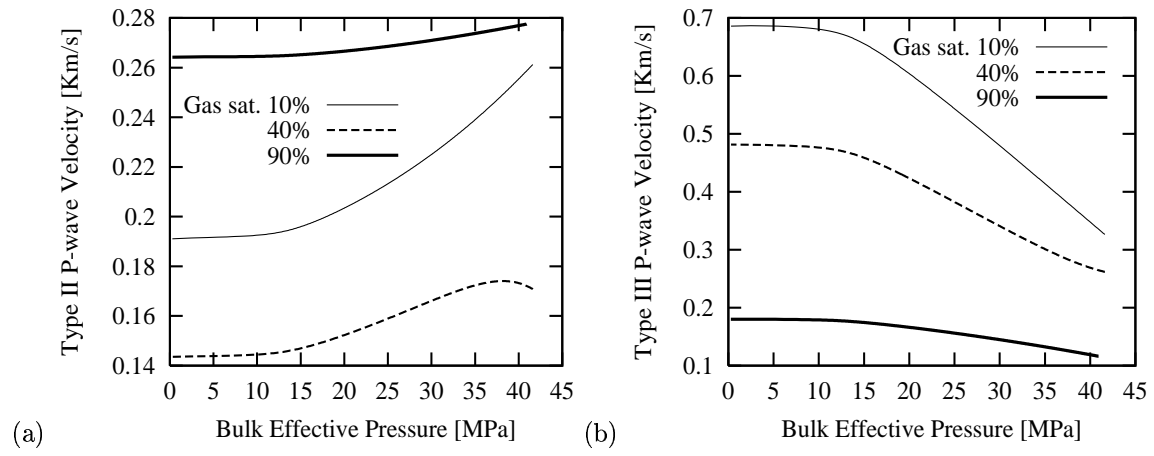




Figure 4

